Improving Mathematics Education

Resources for Decision Making

NATIONAL RESEARCH COUNCIL

Improving Mathematics Education: Resources for Decision Making

Committee on Decisions That Count

Steve Leinwand and Gail Burrill, editors

Mathematical Sciences Education Board
Center for Education
Division of Behavioral and Social Sciences and Education
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This report has been reviewed in draft form by individuals chosen for their diverse perspectives and technical expertise, in accordance with procedures approved by the NRC's Report Review Committee. The purpose of this independent review is to provide candid and critical comments that will assist the institution in making its published report as sound as possible and to ensure that the report meets institutional standards for objectivity, evidence, and responsiveness to the study charge. The review comments and draft manuscript remain confidential to protect the integrity of the deliberative process. We wish to thank the following individuals for their review of this report:

Henry L.Alder, University of California, Davis
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Although the reviewers listed above have provided many constructive comments and suggestions, they were not asked to endorse the conclusions or recommendations nor did they see the final draft of the report before its release. The review of this report was overseen by Edward A.Silver, University of Michigan, and William G.Howard, Jr., Independent Consultant, Scottsdale, Arizona. Appointed by the National Research Council, they were responsible for making certain that an independent examination of this report was carried out in accordance with institutional procedures and that all review comments were carefully considered. Responsibility for the final content of this report rests entirely with the authoring committee and the institution.

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Introduction

> The mathematics students need to learn today is not the same mathematics that their parents and grandparents needed to learn. When today's students become adults, they will face new demands for mathematical proficiency that school mathematics should attempt to anticipate. Moreover, mathematics is a realm no longer restricted to a select few. *All young Americans must learn to think mathematically, and they must think mathematically to learn.* (*Adding It Up,* p. 1)

During the late 1980s and 1990s, events such as publication of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards* (NCTM, 1989), the formation of the National Education Goals Panel, and the call for high expectations for all students spurred demands for the improvement of mathematics teaching and learning. As educators struggled to implement changes or to understand what change might mean in their own systems, the Mathematical Sciences Education Board (MSEB) recognized the need to provide support for those involved in this struggle. So, the Board began work on a document that would provide guidance for educators and other stakeholders making crucial decisions regarding their mathematics programs.

During this time, a variety of groups and organizations produced reports about research findings and policy recommendations relating to mathematical content, effective teaching and learning, and assessment. To ensure that this array of information reached the field in a coherent fashion, MSEB directed its initiative toward helping educators understand how to use these new resources to address key issues related to improving mathematics education. The result *Improving Mathematics Education: Resources for Decision Making,* summarizes the major recommendations in eight documents related to mathematics education and provides an analysis of overlapping recommendations.

To find potential publications, the committee contacted representatives of national mathematics organizations, searched the Web for publications, and considered recent publications from the National Research Council (NRC), the

Department of Education, and other education organizations. The committee was looking for documents that were national in scope, addressed more than one aspect related to K–12 mathematics, and were accessible to educators working to improve mathematics education.

Eight documents form the basis for this report. The following documents are reviewed:

- *Principles and Standards for School Mathematics*
- *Adding It Up: Helping Children Learn Mathematics*
- *How People Learn: Brain, Mind, Experience, and School*
- *Before It's Too Late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21st Century*
- *Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium*
- *The Mathematical Education of Teachers*
- *High Stakes: Testing for Tracking, Promotion, and Graduation*
- *Every Child Mathematically Proficient: An Action Plan of the Learning First Alliance*

The authoring groups for the reports in *Improving Mathematics Education* represent a range of constituencies and viewpoints. Four of the reports were produced by the National Research Council, whose report process is characterized by the deliberate inclusion of balanced viewpoints in committee composition and report review. A fifth report, *Principles and Standards for School Mathematics,* was produced by the National Council of Teachers of Mathematics, which used Association Review Groups to gather input to the report and reactions to a draft version. These Association Review Groups were convened by organizations representing different facets of mathematics and mathematics education. The NCTM process of seeking and incorporating diverse input was reviewed by an NRC committee and judged to be exemplary. An open process of public hearings and testimony led to a sixth report, which was produced by the Glenn Commission a body appointed by the United States Department of Education. After a lengthy process of review and commentary, *The Mathematics Education of Teachers* report was produced by the Conference Board on Mathematical Sciences, an organization of professional mathematical associations. The Learning First Alliance report was produced by a group that represents a broad consortium of public policy groups concerned about mathematics education.

This collection of works represents a variety of perspectives and opinions within the mathematics community but, because there are many and substantially different perspectives, it does not necessarily represent a full range of views. Each report represents a negotiated consensus achieved through the consideration of diverse perspectives.

In addition to having different perspectives on improving mathematics education the eight documents differ in length, potential audience, and scope. For example, the document produced by the Learning First Alliance is not comparable in length or scope to the document produced by the National Council of Teachers of Mathematics. It is, however, significant in at least three ways: (1) it was a significant attempt by a group of those outside of the mathematics education community to address important issues related to improving what happens in mathematics classrooms; (2) it suggested steps the organizations represented by the Alliance might take in order to realize their recommendations; and (3) it served as resource for other groups that built on the thinking in the Learning First Alliance document to produce their own. As another example, the document produced by the Conference Board on Mathematical Sciences and published by the American Mathematics Society is content focused and primarily addresses university mathematics faculty who are in some way responsible for the mathematics preparation of prospective teachers. On the other hand, the document on teacher education published by the NRC is targeted at the wider audience of those who are responsible for preparing teachers. It describes in general the problems and issues related to teacher education and the teaching of science, mathematics, and technology, and it makes policy recommendations that call for restructuring teacher preparation and professional development programs.

Improving Mathematics Education has been designed to help inform stakeholders about the decisions they face, to point to recent research findings, and to provide access to the most recent thinking of experts on issues of national concern in mathematics education. The essence of the report is that information is available to help those charged with improving student achievement in mathematics. The documents cited above can guide those who make decisions about content, learning, teaching, and assessment.

The report is organized around five key questions:

- What should we teach, given what we know and value about mathematics and its roles?
- How should we teach so children learn, given what we know about students, mathematics, and how people learn mathematics?
- What preparation and support do teachers need?
- How do we know whether what we are doing is working?
- What must change?

Each of the five main chapters in this report considers a key area of mathematics education and describes the core messages of current publication (s) in that area. To maintain the integrity of each report's recommendations, we used direct quotes and the terminology defined and used in that report. If the wording or terminology seems to need clarification, the committee refers the reader directly to the original document. Because these areas are inter

dependent, the documents often offer recommendations related to several different areas. While the individual documents are discussed under only one of the components in *Improving Mathematics Education,* the reader should recognize that each document may have a broader scope. In general, the references in this report should serve as a starting point for the interested reader, who can refer to the original documents for fuller discussions of the recommendations and, in some cases, suggestions for implementation. *Improving Mathematics Education* is designed to help educators build a critical knowledge base about mathematics education, recognizing that the future of the nation's students is integrally intertwined with the decisions we make (or fail to make) about the mathematics education they receive.

2

What Should We Teach?

> We live in a time of extraordinary and accelerating change. New knowledge, tools, and ways of doing and communicating mathematics continue to emerge and evolve…. The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase. (*Principles and Standards for School Mathematics,* p. 4)

The mathematical knowledge necessary to succeed in this changing world is tied to what is taught in schools. The core of a mathematics program is its curriculum—what is taught to whom and when. Careful sequencing of mathematical ideas can build understanding and sense making about important mathematical topics. To help educators consider their curriculum, we offer three questions, then suggest a resource generated at the national level that can provide advice for states, districts, and schools formulating their own plans in response to these questions:

- What mathematics should all students know and be able to do?
- What mathematics should be taught at what grade levels?
- What mathematics is important for students in the 21st century?

RESOURCE AVAILABLE

- *Principles and Standards for School Mathematics,* developed by the National Council of Teachers of Mathematics, 2000.

OVERVIEW OF THE RESOURCE

The National Council of Teacher of Mathematics (NCTM) intends its *Principles and Standards for School Mathematics* to be a "resource and guide

for all who make decisions that affect the mathematics education of students in pre-kindergarten through grade 12" (p. ix). NCTM is an organization of over 110,000 mathematics educators concerned with pre-K–12 mathematics education. This update of the NCTM's three previously developed sets of standards for curriculum, teaching, and assessment is intended to establish a curriculum framework to bring focus and coherence to K–12 mathematics. The document was developed through an extensive and inclusive process that engaged a wide spectrum of experts on issues concerning mathematics education. As such, *Principles and Standards* represents a negotiated position about appropriate content for school mathematics to which educators should give careful consideration.

The developers offer the standards as a guide for ensuring quality, developing goals, and promoting change by suggesting common language, examples, and recommendations to engage people at state, provincial and local levels in conversations about mathematics education. The document is intended to (p. 6):

- Set forth a comprehensive and coherent set of goals for mathematics for all students that will orient curricular, teaching, and assessment efforts.
- Serve as a resource for teachers, education leaders, and policymakers to use in examining and improving the quality of mathematics instructional programs.
- Guide the development of curricular frameworks, assessments, and instructional materials.
- Stimulate ideas and ongoing conversations about how best to help students gain a deep understanding of important mathematics.

Principles and Standards is built on the following vision (p. 5):

> In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed. NCTM challenges the assumption that mathematics is only for the select few. On the contrary, everyone needs to understand mathematics. All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence.

To fulfill this vision, the document describes what mathematics in pre-K–12 school programs should look like including how mathematical ideas should be developed across five content areas and five process domains. The standards present a deeper look at the mathematics within each of four grade-level bands, pre-K–2, 3–5, 6–8, and 9–12; they also suggest how mathematics should grow

across the grades. The examples were chosen to illustrate these ideas and to portray teaching practices that will support all students in learning such mathematics.

RECOMMENDATIONS MADE IN THE REPORT

Essentially the recommendations made in *Principles and Standards* are the principles and standards themselves. The document offers six overarching themes—the "principles for schools mathematics" —to provide a lens for considering decisions about the content and character of school mathematics (p. 11):

- **Equity.** Excellence in mathematics education requires equity—high expectations and strong support for all students.
- **Curriculum.** A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.
- **Teaching.** Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- **Learning.** Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- **Assessment.** Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
- **Technology.** Technology is essential in teaching and learning mathematics: It influences the mathematics that is taught and enhances students' learning.

The mathematical content of the curriculum is organized into five major content strands for school mathematics (numbers and operations, algebra, geometry, measurement, and data analysis and probability) and five major process domains (problem solving, reasoning and proof, communication, connections, and representations). The five strands describe the content students should learn, and the five domains describe the processes through which students learn mathematics and demonstrate the mathematics they have learned. The heart of *Principles and Standards* elaborates on the mathematical expectations, instructional strategies, and assessment practices that guide the effective and coherent implementation of these standards at the four grade bands.

Highlighted recommendations include the following:

- "Teachers must help students be confident, engaged mathematics learners." (p. 374)
- "Teacher-leaders support on a day-to-day basis can be crucial to a teacher's work life." (p. 375)
- "Administrators and policymakers must carefully consider the impact of high-stakes assessments on the instructional climate in schools." (p. 377)
- "Families become advocates for education standards when they understand the importance of a high-quality mathematics education for their children." (p. 378)
- "Conceptual understanding is an important component of proficiency." (p. 20)

Putting this vision into action requires thoughtful and ongoing consideration of the following questions:

- "How can all students have access to high-quality mathematics education?" (p. 368)
- "Are good instructional materials chosen, used, and accepted?" (p. 369)
- "How can teachers learn what they need to know?" (p. 370)
- "Do all students have time and the opportunity to learn?" (p. 371)
- "Are assessments aligned with instructional goals?" (p. 372)
- "Is technology supporting learning?" (p. 372)

ACTIONS EDUCATORS MIGHT CONSIDER

As a resource for high-quality mathematics programs, *Principles and Standards* can be used to guide analysis and decision making about current programs and evolving program components. To consider what mathematics all students should know and at what grade bands it should be taught, educators and policymakers might do the following:

- Compare their curricular guidelines and frameworks with the standards and expectations delineated in *Principles and Standards* and examine the differences.
- Compare the content and format of classroom, district, and state assessments in current use with the vision of curriculum described in these documents and consider the implications of mismatches.
- Disaggregate enrollment and achievement data to analyze patterns of equity of opportunity and equity of outcome and develop plans and programs that respond to any differences that are found.
- Develop and implement professional development opportunities to ensure that all teachers of mathematics have the mathematical and pedagogical knowledge and skill needed to implement a curriculum that will meet the needs of their students.

- Develop programs and policies to ensure that school, district, and state administrators who are responsible for shaping instructional programs provide the human and material resources necessary for their implementation at the school, district, and state levels.
- Develop programs and policies to ensure that all those involved in preservice preparation of teachers and responsible for in-service or professional development programs have a sufficient understanding of content, curriculum, teaching, and learning.

Discussion about these suggestions can be found in some of the documents discussed in later chapters of this report. For example, both *Before It's Too Late* and *Adding It Up* provide recommendations about teacher preparation.

3

How Should We Teach?

How should we teach so students learn? What students learn is related to how they learn. What do we know about how students process ideas and how they put them together to make sense of the mathematics they are studying? How is this different from current practice? As educators decide what mathematics programs they want for their students, they should consider not only what content is important but also what research can tell us about how students learn and how this should inform the curriculum they put in place and the instructional processes used to deliver that curriculum. These questions should drive decision making:

- How do students learn mathematics?
- What are the implications of what we know about how students learn for curriculum and instruction?
- What is the nature of teaching practice supported by research in cognitive science?

RESOURCES AVAILABLE

- *Adding It Up: Helping Children Learn Mathematics,* developed by the National Research Council's Mathematics Learning Study Committee, 2001.
- *How People Learn: Brain, Mind, Experience, and School,* developed by the National Research Council's Committee on Developments in the Science of Learning and the Committee on Learning Research and Educational Practice, 2000.

OVERVIEW OF THE RESOURCES

These two resources address issues of research related to student learning. *Adding It Up* sets forth what we know about children's learning of mathematics,

particularly number, in grades pre-K–8, as well as the implications this knowledge has for teaching. *How People Learn* calls on recent findings in brain research to address issues of learning in general. In particular, it describes how skill and understanding in key subjects are acquired and discusses our growing knowledge about complex reasoning and problem solving.

Adding It Up: Helping Children Learn Mathematics

Adding It Up is a research-based examination of pre-K–8 mathematics that focuses on *what* the mathematical content is with which students must develop proficiency, *how* instruction can help students develop this proficiency, and, most significant, the research that undergirds *why* these positions are taken. The report was developed by the Mathematics Learning Study Committee of the National Research Council. It arose out of concerns on the part of the National Science Foundation and the U.S. Department of Education about the shortage of reliable information on the learning of mathematics that could guide best practice.

The developers of *Adding It Up* based their findings and recommendations on research that is "*relevant* to important educational issues, *sound* in shedding light on the questions it sets out to answer, and *generalizable* in that it can be applied to circumstances beyond those of the study itself." They also "looked for multiple lines of research that *converge* on a particular point" (p. 3). Using the analysis of this research, much of which is synthesized in the report, the committee states that *"All young Americans must learn to think mathematically, and they must think mathematically to learn"* (p. 16).

"Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics" the report chooses the phrase *mathematical proficiency* to capture "what it means for anyone to learn mathematics successfully." Mathematical proficiency is then defined as having five interwoven and interdependent strands (p. 5):

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations;
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- *strategic competence*—ability to formulate, represent, and solve mathematical problems;
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification; and
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

While the report focuses on curriculum and learning, some of the discussion relates to instruction and to issues of professional development. A key message is that the report endorses no single instructional approach but contends that "instruction needs to configure the relations among teachers, students, and mathematics in ways that promote the development of mathematical proficiency. Under this view, significant instructional time is devoted to developing concepts and methods; carefully directed practice, with feedback, supports learning. Discussions build students' thinking, attend to relationships between problems and solutions and to the nature of justification and mathematical argument as the strands of proficiency grow in a coordinated, interactive fashion" (p. 11).

Finally, explaining why so much of the report focuses on the domain of number, it notes that "most of the controversy over how and what mathematics should be taught in elementary and middle school revolves around number" (p. 20), including questions such as the following:

- Should children learn computational methods before they understand the concepts involved?
- Should they be introduced to standard algorithms for arithmetic computation, or should they be encouraged to develop their own algorithms first?
- How proficient do children need to be at paper-and-pencil arithmetic before they are taught algebra and geometry?

Thoughtful discussion about these and similar controversial questions is provided in this report, which considers the mathematical knowledge children bring to school and how students develop proficiency with numbers and in other mathematical areas. The report also discusses teaching for mathematical proficiency, describes instruction as "interactions among teachers and students around content" (p. 313), and outlines what it takes to be proficient at mathematics teaching.

Adding It Up emphasizes two points:

- "Our experiences, discussions, and review of the literature have convinced us that school mathematics demands substantial change" (p. 407); and
- "…[T]hroughout the grades from pre-K through 8 all students can and should be mathematically proficient" (p. 409).

Adding It Up comes to the following conclusion:

> School mathematics in the United States does not now enable most students to develop the strands of mathematical proficiency in a sound fashion. Proficiency for all demands fundamental changes be made concurrently in curriculum,

> instructional materials, classroom practice, teacher preparation, and professional development. These changes will require continuing, coordinated action on the part of policymakers, teacher educators, teachers, and parents. Although some readers may feel that substantial advances are already being made in reforming mathematics teaching and learning, we find real progress toward mathematical proficiency to be woefully inadequate, (p. 409)

These observations lead to five general recommendations and a series of specific recommendations that detail the policies and practices needed if all children are to become mathematically proficient. The five general recommendations are as follows (pp. 409–410):

- "The integrated and balanced development of all five strands of mathematical proficiency should guide the teaching and learning of school mathematics. Instruction should not be based on extreme positions that students learn, on the one hand, solely by internalizing what a teacher or book says or, on the other hand, solely by inventing mathematics on their own.
- Teachers' professional development should be high quality, sustained, and systematically designed and deployed to help all students develop mathematical proficiency. Schools should support, as a central part of teachers' work, engagement in sustained efforts to improve their mathematics instruction. This support requires the provision of time and resources.
- The coordination of curriculum, instructional materials, assessment, instruction, professional development, and school organization around the development of mathematical proficiency should drive school improvement efforts.
- Efforts to improve students' mathematics learning should be informed by scientific evidence, and their effectiveness should be evaluated systematically. Such efforts should be coordinated, continual, and cumulative.
- Additional research should be undertaken on the nature, development, and assessment of mathematical proficiency."

Among the 58 specific recommendations made to help move the nation toward the change needed in school mathematics, *Adding It Up* urges the following with respect to student learning and its relation to curriculum:

- "Mathematics programs in the early grades should make extensive use of appropriate objects, diagrams, and other aids to ensure that all children understand and are able to use number words and the base-10 properties of numerals, that all children can use the language of

quantity (hundreds, tens, and ones) in solving problems, and that all children can explain their reasoning in obtaining solutions." (p. 412)
- "Instructional materials and classroom teaching should help students learn increasingly abbreviated procedures for producing number combinations rapidly and accurately without always having to refer to tables or other aids." (p. 413)
- "For addition, subtraction, multiplication, and division, all students should understand and be able to carry out an algorithm that is general and reasonably efficient." (p. 414)
- "The basic ideas of algebra as generalized arithmetic should be anticipated by activities in the early elementary grades and learned by the end of middle school." (p. 419)
- "Problem solving should be the site in which all the strands of mathematics proficiency converge. It should provide opportunities for students to weave together the strands of proficiency and for teachers to assess students' performance on all of the strands." (p. 421)

With respect to assessment, *Adding It Up* recommends (pp. 423–424):

- Assessment, whether internal or external, should be focused on the development and achievement of mathematical proficiency.
- The results of each external assessment should be reported so as to provide feedback useful for teachers and learners rather than simply a set of rankings.

With respect to instruction, *Adding It Up* recommends the following:

- "A significant amount of class time should be spent in developing mathematical ideas and methods rather than only practicing skills." (p. 425)
- "Questioning and discussion should elicit students' thinking and solution strategies and should build on them, leading to greater clarity and precision." (p. 426)
- "Discourse should not be confined to answers only but should include discussion of connections to other problems, alternative representations and solution methods, the nature of justification and argumentation, and the like." (p. 426)
- "In all grades of elementary and middle school, any use of calculators and computers should be done in ways that help develop all strands of students' mathematical proficiency." (p. 427)

With respect to teacher preparation, the report makes the following recommendations (p. 429):

- To provide a basis for continued learning by teachers, their preparation to teach, their professional development activities, and the instructional

materials they use should engage them, individually and collectively, in developing a greater understanding of mathematics and of student thinking, and in finding ways to put that understanding into practice. All teachers, whether preservice or inservice, should engage in inquiry as part of their teaching practice.

- Teachers of grades pre-K to 8 should have a deep understanding of the mathematics of the school curriculum and the principles behind it.
- Mathematics specialists—teachers who have special training and interest in mathematics—should be available in every elementary school.

How People Learn: Brain, Mind, Experience, and School

How People Learn is an example-laden review of what is known about learning and its implications for teaching. In the medical profession, research-based practice and revisions to practice based on newer research are the norm. Education needs to develop a similar culture and similar expectations for using research to inform and direct practice. The development of a "science of learning" and the translation of this science into practice can be critical for enhancing educational productivity.

As noted early in the report: "the new science of learning is beginning to provide knowledge to improve significantly people's abilities to become active learners who seek to understand complex subject matter and are better prepared to transfer what they have learned to new problems and settings…. The emerging science of learning underscores the importance of rethinking what is taught, how it is taught, and how learning is assessed" (p. 13). Learning research suggests a need to change current practice; "there are new ways to introduce students to traditional subjects…and that these new approaches make it possible for the majority of individuals to develop a deep understanding of important subject matter" (p. 6).

The scientific achievements synthesized in the report "include a fuller understanding of (1) memory and the structure of knowledge, (2) problem solving and reasoning, (3) the early foundations of learning, (4) regulatory processes that govern learning, including metacognition, and (5) how symbolic thinking emerges from the culture and community of the learner" (p. 14). This overview of the research generates three key findings highlighted in the report:

- "Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom." (p. 14)

- "To develop competence in an area of inquiry, students must (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application." (p. 16)
- "A 'metacognitive' approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them." (p. 18)

The implications of these findings for teaching are summarized similarly:

- "Teachers must draw out and work with the pre-existing understandings that their students bring with them." (p. 19)
- "Teachers must teach some subject matter in depth, providing many examples in which the same concept is at work and providing a firm foundation of factual knowledge." (p. 20)
- "The teaching of metacognitive skills should be integrated into the curriculum in a variety of subject areas." (p. 21)

These findings are then translated into four interrelated attributes of learning environments:

- "Schools and classrooms must be learner centered." (p. 23)
- "To provide a knowledge-centered classroom environment, attention must be given to what is taught (information, subject matter), why it is taught (understanding), and what competence or mastery looks like." (p. 24)
- "Formative assessments—ongoing assessments designed to make students' thinking visible to both teachers and students—are essential. They permit the teacher to grasp the students' preconceptions, understand where the students are in the 'development corridor' from informal to formal thinking, and design instruction accordingly. In the assessment-centered classroom environment, formative assessments help both teacher and students monitor progress." (p. 24)
- "Learning is influenced in fundamental ways by the context in which it takes place. A community-centered approach requires the development of norms for the classroom and school, as well as connections to the outside world, that support core learning values." (p. 25)

Each of the findings, implications and attributes listed above is supported and elaborated upon in great detail in the report. In lieu of specific recommendations, *How People Learn* concludes with a detailed set of summary findings and conclusions and 23 categories of recommended research and development for future research. Recommendations for practice include the following:

- People's ability to transfer what they have learned depends upon a number of factors:

  – "People must achieve a threshold of initial learning that is sufficient to support transfer." (p. 235)
  – "Spending a lot of time ("time on task") in and of itself is not sufficient to ensure effective learning." (p. 235)
  – "Learning with understanding is more likely to promote transfer than simply memorizing information from a text or a lecture." (p. 236)
  – "Knowledge that is taught in a variety of contexts is more likely to support flexible transfer than knowledge that is taught in a single context." (p. 236)
  – "Students develop flexible understanding of when, where, why, and how to use their knowledge to solve new problems if they learn how to extract underlying themes and principles from their learning exercises." (p. 236)
  – "All learning involves transfer from previous experiences." (p. 236)
  – "Sometimes the knowledge that people bring to a new situation impedes subsequent learning because it guides thinking in the wrong direction." (p. 236)

- Teachers need expertise in both subject-matter content and in teaching; need to develop an understanding of the theories of knowledge that guide the subject-matter disciplines in which they work; need to develop an understanding of pedagogy as an intellectual discipline that reflects theories of learning, including knowledge of how cultural beliefs and the personal characteristics of learners influence learning; and need to develop models of their own professional development that are based on lifelong learning, rather than on an "updating" model of learning (p. 242).
- "Computer-based technologies hold great promise both for increasing access to knowledge and as a means of promoting learning" (p. 243).

ACTIONS EDUCATORS MIGHT CONSIDER

As comprehensive summaries of what we currently know about learning in general, about learning mathematics in particular, and about the implications of this knowledge for teaching and creating school environments conducive to learning, *Adding It Up* and *How People Learn* can serve as resources for professional discussions, for course and seminar content, and for guiding future research. What we know about how students learn suggests that educators, researchers, and policy makers can do the following:

- Draw from the extensive findings in these reports to educate parents and the community at large about, and build support for, changes in school mathematics programs.

- Analyze current practice in light of research findings about learning and work to enable more research-based practices in schools and classrooms.
- Support additional research along the lines proposed in Chapter 11 of *How People Learn,* forming teams that combine the expertise of researchers and the wisdom of practitioners. This research should focus on curriculum materials, formative assessment, the use of technology, and on the alignment and effectiveness of professional development programs.
- Analyze, and adjust the curriculum, instructional practices, and assessments that are used, in light of the evidence presented throughout *Adding It Up.*
- Create study groups and other professional development opportunities that bring together teachers, administrators, and teacher educators; use selected chapters from *How People Learn* and *Adding It Up* as catalysts for discussion and for planning improvements.

4

What Preparation and Support Do Teachers Need?

Teachers are the key to improving mathematics education. What teachers know how to do and what they choose to do when delivering instruction in their classrooms determine what content students learn and which students learn that content. The preparation, certification, and ongoing professional development of teachers define what they are able to do with theit students. Two important facets shape teacher preparation: (1) what mathematics. the teachers need to know to teach and (2) how they learn to teach that mathematics. Ensuring that well-prepared and competent teachers are in every mathematics classroom means considering the following questions:

- What should teachers of mathematics know and be able to do?
- How do teachers need to learn this material and these skills?
- What is the nature of coursework that will prepare prospective teachers to be efective teachers of mathematics?
- How can schools and school systems institutionalize a system of ongoing preofessional development?
- How can schools and school systems create a prefessional working environment to make teaching more attractive?

RESOURCES AVAILABLE

- *Before It's Too Late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21 st Century,* chartered by the United States Secretary of Education, 2000.
- *Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium,* development by the National Research Council's Committee on Science and Mathematics Teacher Preparation, 2000.
- *The Mathematical Education of Teachers,* development by the Conference Board of the Mathematical Sciences, 2001

OVERVIEW OF THE RESOURCES

These three resources address the issues of who is going to teach mathematics (and science), how they should be prepared, and how they should be supported professionally once they are in the classroom. *Before It's Too Late* suggests that the lack of well-prepared mathematics and science teachers are a critical national issue. It argues that to improve our students' proficiency in mathematics and science, we must improve mathematics and science teaching. *Educating Teachers* makes the case, based on research, for well-prepared teachers and calls for restructuring teacher preparation and professional development. Finally, *The Mathematical Education of Teachers* describes the mathematics a teacher needs to know and what mathematics programs should look like to deliver that knowledge.

Before It's Too Late

Before It's Too Late is the report of the National Commission on Mathematics and Science Teaching for the 21st Century (the Glenn Commission), which was charged with "(1) reviewing the current state of American K–12 mathematics and science education with a focus on the challenges of teacher recruitment, preparation, retention, and professional growth and (2) articulating the steps needed to strengthen the classroom practice of math and science teachers" (p. 46). *Before It's Too Late* is predicated on four premises:

- "At the daybreak of this new century and millennium, the future well-being of our nation and people depends not just on how well we educate our children generally, but on how well we educate them in mathematics and science specifically." (p. 4)
- "It is abundantly clear from the evidence already at hand that we are not doing the job that we should do—or can do—in teaching our children to understand and use ideas from these fields." (p. 4)
- "The most powerful instrument for change, and therefore the place to begin, lies at the very core of education—*with teaching itself*." (p. 5)
- "Committing ourselves to reach three specific goals—going directly to the issues of quality, quantity, and an enabling work environment—can go far in bringing about the basic changes we need." (p. 5)

Summarizing what happens in most classrooms, the report notes that: "If the core of mathematics and science is about inquiry, then too many of today's mathematics and science classrooms come up short. Students are crippled by content limited to 'What?' They get only a little bit about the 'How?' (or 'How else?') and not nearly enough about the 'Why?' Missing almost entirely is

'Why should I care?' It is hard to imagine that students in these classes are gaining the conceptual and problem-solving skills they need to function effectively as workers and citizens in today's world" (p. 21).

This scenario is contrasted with the following vision of high-quality teaching (p. 22):

- High-quality teaching requires that teachers have a deep knowledge of subject matter.
- The ability to teach, contrary to myth, is not "something you're born with"; it can be learned and refined over time…through training, mentoring, collaboration with peers, and practice.
- The process of inquiry, not merely "giving instruction", is the very heart of what teachers do.
- A good science or mathematics teacher encourages students to try new possibilities, to venture possible explanations, and to follow them to their logical conclusions.
- High quality teaching fosters healthy skepticism.
- High quality teaching has the deepest respect for students as persons, and builds on strengths, rather than trying to stamp out weaknesses.
- Teaching is grounded in a careful and thorough alignment of curriculum, assessment, and high standards for student learning.
- Practice is continually reshaped by supportive institutional structures, such as professional development, continuing education, and the effective use of technology.
- Effectiveness is evaluated by student performance and achievement.

Asking why high-quality teaching is not universal, the report makes the following observation:

> For teachers to deliver high-quality teaching, they must be empowered to do so. Generating this kind of teaching means that school boards, administrators, parents, and policymakers must be willing to stand up for teachers as the primary drivers of student achievement. Teachers must be given the time they need within the school day to keep up with new developments in their fields, teaching aids, materials, and technology. Teachers must be encouraged to contribute knowledge back to their disciplines. They need the time and the feedback necessary to reflect on their teaching, so they can get better at it. Teacher empowerment also means according teachers the respect they deserve for their judgments about learning, rewarding their professionalism, and yes, paying them what they are worth. (p. 23)

Before It's Too Late proposes three goals or recommendations and sets forth 14 action strategies by which to implement the goals:

- "Establish an ongoing system to improve the quality of mathematics and science teaching in grades K–12" by (p. 24):

  – Immediately conducting a full needs assessment.
  – Establishing summer institutes.
  – Developing school- and district-level inquiry groups.
  – Providing leadership training.
  – Creating an Internet portal for teachers.
  – Forming a nongovernmental Coordinating Council for Mathematics.
  – Initiating reward and incentive programs.

- "Increase significantly the number of mathematics and science teachers and improve the quality of their preparation" by (p. 29):

  – Identifying exemplary models of teacher preparation.
  – Finding ways to attract additional qualified candidates into teaching.
  – Creating 15 Mathematics and Science Teaching Academies.

- "Improve the working environment and make the teaching profession more attractive for K–12 mathematics and science teachers" by (p. 32):

  – Instituting focused induction programs to acclimate beginning teachers to the profession.
  – Developing district/business partnerships to help create more professional working environments.
  – Providing incentives—in the form of cash awards, salary increases, support for further education, or community-wide recognition to encourage teachers to remain in teaching.
  – Making salaries—especially for mathematics and science teachers—more competitive.

Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium

Educating Teachers was developed by a National Research Council committee charged with identifying "critical issues emerging from existing practices and policies for teacher preparation" (p. 27).

The report includes a detailed analysis of the problems and issues relating to teacher education and the teaching of science, mathematics, and technology; a

summary of current recommendations concerning these issues; and recommendations for a systemic approach to improving teacher education. Among the teacher education issues raised (pp. 31–34), the report notes the following:

- Research is demonstrating that good teaching does matter.
- Many of the nation's teachers are not adequately prepared to teach mathematics and science using standards-based approaches and in ways that bolster student learning and achievement.
- The preparation of beginning teachers often does not meet the needs of the modern classroom.
- Accreditation standards and licensing examinations often do not reflect recent changes in expectations for teachers or for students.
- Professional development for continuing teachers too often consists of a patchwork of courses, curricula, and programs and may do little to enhance teachers' content knowledge or the techniques and skills they need to teach science and mathematics effectively.

Examining results from emerging research, the study committee concluded that a teacher must have conceptual understanding of mathematics. It also argues that pedagogical content knowledge is an important subset of content knowledge (p. 119).

The report also presents a broad set of issues related to teaching in today's classrooms (pp. 35–39) and compares teaching with other professions. The report indicates that the teaching profession falls far short of the expectations, rewards, and working conditions found in other professions. Significant systemic improvements are needed to provide the following:

- Access to adequate career advice from college-level faculty in the sciences, mathematics, or engineering.
- Rigorous and appropriate content courses for prospective teachers.
- Oversight of teacher education programs by professional organizations.
- A continuum of professional development.
- Adequate mentoring of new employees; targeted professional development programs; encouragement and incentives for continuing education within the profession; and expectations for credentialing of professionals.
- The involvement of teachers in decision- and policy-making.

The vision for improving teacher education and the teaching profession that emerges in the report is built upon six guiding principles (p. 88):

- "The improvement of teacher education and teaching in science, mathematics, and technology should be viewed as a top national priority;

- Teacher education in science, mathematics, and technology must become a career-long process. High quality professional development programs that include intellectual growth as well as the upgrading of teachers' knowledge and skills must be expected and essential features in the careers of all teachers;
- Through changes in the rewards for, incentives for, and expectation of teachers, teaching as a profession must be upgraded in status and stature to the level of other professions;
- Both individually and collectively, two- and four-year colleges and universities must assume greater responsibility and be held more accountable for improving teacher education;
- Neither the higher education nor the K–12 communities can successfully improve teacher education as effectively in isolation as they can by working closely together. Collective, fully integrated efforts among school staff and administrators in individual schools and districts, teacher unions, faculty and administrators of higher education, policymakers, from local colleges and universities, and parents are essential for addressing these issues; and
- Many more scientists, mathematicians, and engineers, must become well informed enough to be involved with local and national efforts to provide the appropriate content knowledge and pedagogy of their disciplines to current and future teachers."

Educating Teachers offers three general recommendations (p. 109):

- "Teacher education in science, mathematics and technology be viewed as a continuum of programs and professional experiences that enables individuals to move seamlessly from college preparation for teaching to careers in teaching these subject areas;
- Teacher education be viewed as a career-long process that allows teachers of science, mathematics, and technology to acquire and regularly update the content knowledge and pedagogical tools needed to teach in ways that enhance student learning and achievement in these subjects; and
- Teacher education also be structured in ways that allow teachers to grow individually in their profession and to contribute to the further enhancement of both teaching and their disciplines."

These general recommendations are reinforced by a set of specific recommendations for government, for collaboration between institutions of higher education and the K–12 community, for the higher education community, for the K–12 education community, and for professional and disciplinary organizations. As a whole, these recommendations advocate building a far more coherent, collaborative and professional system of teacher education. The report highlights the following specific activities, among others:

- Building partnerships and other forms of collaboration between institutions of higher education and K–12 school districts to improve teacher education.
- Increasing reliance on the professional development school model of teacher preparation.
- Establishing collaborative beginning and experienced teacher support programs.
- Restructuring which organizations have primarily responsibility for the various components of teacher education.
- Enhancing links between research and practice at all levels of the system.

The Mathematical Education of Teachers

The Mathematical Education of Teachers is designed "to be a resource for mathematics faculty and other parties involved in the education of mathematics teachers. It is a distillation of current thinking on curriculum and policy issues affecting the mathematical education of teachers, with the goal of stimulating efforts on individual campuses to improve programs for prospective teachers". It is also meant to stimulate "efforts on individual campuses to improve programs for prospective teachers" (p. xi). Accordingly, the report "calls for a rethinking of the mathematical education of prospective teachers within mathematical sciences departments at U.S. two- and four-year colleges and universities. It offers principles to assist departments in this process, along with specific suggestions for mathematics courses for prospective teachers." Moreover, the report "seeks to convince faculty that there is more intellectual content in school mathematics instruction than most realize, content that teachers need to understand well" (p. 3).

The report has been prepared by the Conference Board of the Mathematical Sciences, an umbrella organization of 16 professional societies, all of which have as one of their primary objectives the increase or diffusion of knowledge in one or more of the mathematical sciences. *The Mathematical Education of Teachers* connects the mathematical aspects of teacher preparation with the mathematical content teachers are expected to teach through a set of 11 broad recommendations (and accompanying specifics) for effectively preparing all teachers of mathematics.

As noted in the preface:

> The mathematical knowledge needed for teaching is quite different from that required by college students pursuing other mathematics-related professions. Prospective teachers need a solid understanding of mathematics so that they can teach it as a coherent, reasoned activity and communicate its elegance

> and power. Mathematicians are particularly qualified to teach mathematics in the connected, sense-making way that teachers need. For maximum effectiveness, the design of this instruction requires collaboration between mathematicians and mathematics educators and close connections with classroom practice, (p. xi)

These themes are played out in three sets of recommendations for mathematical content for elementary, middle, and high school teachers. In addition to specific content recommendations at each level, the report's supporting commentaries provide insights into preparing teachers. For example, in the elementary grades chapter, it is suggested that

> The key to turning even poorly prepared prospective elementary teachers into mathematical thinkers is to work from what they *do* know—the mathematical ideas they hold, the skills they possess, and the contexts in which these are understood—so they can move from where they are to where they need to go. For their instructors, this requires learning to understand how their students think. The disciplinary habits of abstraction and deductive demonstration, characteristic of the way professional mathematicians present their work, have little to do with the ways each of us initially enters the world of mathematics, that is, experientially, building our concepts from action. And this is where mathematics courses for elementary school teachers must begin, first helping teachers make meaning for the mathematical objects under study— meaning that often was not present in their own elementary educations—and only then moving on to higher orders of generality and rigor, (p. 17)

Similarly, in the middle grades chapter, readers are informed that:

> One way to develop meaning in algebra is to highlight the manner in which algebra is generalized arithmetic, a language that encodes properties of arithmetic operations. A somewhat different way to think of algebra is as an extension of quantitative reasoning in arithmetic situations. If arithmetic word problems are solved by focusing on the quantities in a problem and determining relationships among these quantities before assigning any numerical values to the quantities, it is a reasonable next step to assign variables rather than numbers. Assigning variables to the quantities and setting up equations representing the relationships is then a formalization of reasoning quantitatively about the problem. However, this formalization is not always an easy one. Prospective teachers

> need practice on solving problems situated in realistic contexts through this type of analysis, which can also help them develop a deeper appreciation of the important role variables play in algebra. (p. 31)

In the high school chapter, it is suggested that:

> Algebraic connections between high school and college courses can be an explicit focus of the capstone sequence for teachers. For example, this sequence could profitably examine the historical evolution of key concepts in number theory and algebra and it could trace the development of key number and algebra ideas from early secondary school through contemporary applications. It could examine the crucial role of algebra in use of computer tools like spreadsheets and the ways that computer algebra system might be useful in exploring algebraic idea. Each facet in such a capstone treatment of number and algebra would provide teachers with insight into the structure of high school mathematics, its uses in science and technology or in the workplace, and the conceptual difficulties in learning number and algebraic concepts. (p. 41)

Clearly, this report calls for substantive reform of both the content and the approach to mathematical coursework for prospective teachers as well as a renewed commitment on the part of the mathematics faculty to meeting their mathematical needs.

The Mathematical Education of Teachers makes 11 broad recommendations:

- "Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach." (p. 7)
- "Although the quality of mathematics preparation is more important than the quantity, the following amount of mathematics coursework for prospective teachers is recommended:

  – Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics;
  – Prospective middle grades teachers of mathematics should be required to take at least 21 semester hours of mathematics, that includes at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers; and
  – Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in

mathematics, that includes a 6-hour capstone course connecting their college mathematics courses with high school mathematics." (P. 7)

- "Courses on fundamental ideas of school mathematics should focus on a thorough development of basic mathematical ideas. All courses designed for prospective teachers should develop careful reasoning and mathematical "common sense" in analyzing conceptual relationships and in solving problems." (p. 8)
- "Along with building mathematical knowledge, mathematics courses for prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching." (p. 8)
- "Teacher education must be recognized as an important part of mathematics departments' mission at institutions that educate teachers. More mathematicians should consider becoming deeply involved in K– 12 mathematics education." (p. 8)
- "The mathematical education of teachers should be seen as a partnership between mathematics faculty and mathematics education faculty." (p. 9)
- "There needs to be greater cooperation between two-year and four-year colleges in the mathematical education of teachers." (p. 9)
- "There need to be more collaborations between mathematics faculty and school mathematics teachers." (p. 10)
- "Efforts to improve standards for school mathematics instruction as well as for teacher preparation, accreditation, and teacher certification, will be strengthened by the full-fledged participation of the academic mathematics community." (p. 10)
- "Teachers need the opportunity to develop their understanding of mathematics and its teaching throughout their careers, through both self-directed and collegial study, and through formal coursework." (p. 10)
- "Mathematics in middle grades (grades 5–8) should be taught by mathematics specialists." (p. 11)

ACTIONS EDUCATORS MIGHT CONSIDER

After a period of collaborative planning and preparation—following the steps proposed in *Before It's Too Late* (pp. 38–1) and the detailed recommendations in *Educating Teachers* (pp. 109–130) —teachers, teacher educators, administrators, and policymakers concerned with how teachers learn the content knowledge and skills to teach might do the following:

- Establish partnerships between K–12 communities and higher education institutions to share the responsibility for preparing and

supporting teachers, using mechanisms like professional development schools.

- Suggest that the research conducted by universities include the development and execution of peer-reviewed studies that focus on (1) ways to improve teacher education, (2) the art of teaching, and (3) how people of different ages learn.
- Work for adequate funding for ongoing professional development.
- Provide teachers with significant professional development opportunities to improve their teaching through in-depth study in the context of inquiry groups and summer institutes.
- Establish induction programs to ensure that new teachers receive the support they need to be effective.
- Establish programs and policies that develop teacher leaders who facilitate the continuous learning of their colleagues.
- Ensure that teachers and other school staff have electronic and other forms of access to the ever-expanding knowledge base about teaching.
- Work to develop career-long incentives and rewards for effective teachers that encourage them to remain in teaching and to continually upgrade their skills.
- Encourage students to consider teaching as a profession.

In addition, using *The Mathematical Education of Teachers* as a guide, faculty members, administrators, and policymakers—particularly at institutions of higher education—who are concerned with what mathematics content teachers should know can:

- Ensure that the courses offered by two- and four-year colleges and universities provide teachers and prospective teachers with strong exposure to appropriate content and model the kinds of pedagogical approaches appropriate for teaching that content.
- Review certification requirements to ensure sufficient and appropriate coursework in mathematics, particularly courses that focus on teaching and understanding the fundamental ideas of school mathematics.
- Foster greater collaboration among mathematicians, mathematics education faculty, and K–12 classroom teachers when designing and implementing programs for prospective teachers.
- Advocate the use of mathematics specialists for all mathematics instruction in grade 5 and beyond.

5

How Do We Know That What We Are Doing Is Working?

Assessment has always been a critical element in education, used by classroom teachers, schools, districts, and (increasingly) states to determine what students know and what they are able to do. Its pervasiveness, political importance, and potential influence over student learning make it a powerful tool for change. Understanding how assessment operates, however, requires considering the entire range of assessments—formative classroom assessment, classroom tests, state and local tests, college entrance and placement practices, tests for teacher certification—and the context in which each of these is used. The interpretation and application of specific assessment tools is of critical concern for educators, and for researchers as well as for families and the community. The following questions should drive decisions made about assessment:

- How can we assess the extent to which what we are doing is working?
- How can student assessment yield accurate information regarding student achievement in mathematics?
- How can large-scale student assessments be used fairly and appropriately?
- How can critical decisions about tracking, promotion, and graduation be made on the basis of student assessments?

RESOURCE AVAILABLE

- *High Stakes: Testing for Tracking, Promotion, and Graduation,* developed by the National Research Council's Committee on Appropriate Test Use, 1999.

OVERVIEW OF THE RESOURCE

High Stakes is a report by the National Research Council developed in response to a congressional request for such a study and for recommendations "on appropriate methods, practices, and safeguards to ensure that (a) existing and new tests that are used to assess student performance are not used in a discriminatory manner or inappropriately for student promotion, tracking, or graduation; and (b) existing and new tests adequately assess student reading and mathematics comprehension" (p. 1).

The report serves as a primer for the sensible use of high-stakes tests—capitalizing on their positive characteristics and minimizing their negative aspects. As noted in the introduction:

> Most people seem to agree that America's public schools are in need of repair. How to fix them has become a favorite topic of policymakers, and for many the remedy includes increased reliance on the testing of students. The standards-based reform movement, for example, is premised on the idea of setting clear, high standards for what children are supposed to learn and then holding students—and often educators and schools—to those standards.
>
> The logic seems clear: Unless we test students' knowledge, how will we know if they have met the standards? And the idea of accountability, which is also central to this theory of school reform, requires that the test results have direct and immediate consequences: A student who does not meet the standard should not be promoted, or awarded a high school diploma. This report is about the appropriate use of tests in making such high-stakes decisions about individual students, (p. 13)

High Stakes considers what constitutes appropriate use of tests in making teaching, promotion, and graduation decisions affecting individual students and emphasizes three criteria for judging the appropriateness of a particular test (p. 23):

- **"Measurement validity.** Is the test appropriate for a particular purpose? Is there evidence that the constructs to be measured are relevant in making a decision? Does the test measure those constructs? Is it confounded with other constructs that are not relevant to the decision? Is the test reliable and accurate?
- **Attribution of cause.** Does a student's performance on a test reflect knowledge and skills based on appropriate instruction, or is it attributable to poor instruction? Or is it attributable to factors such as language barriers or disabilities that are irrelevant to the construct being measured?

- **Effectiveness of treatment.** Does performance on the test lead to placements or other decisions that are educationally beneficial and well matched to the student's needs?"

Based on these criteria, the reader is reminded that "blanket criticism of tests is not justified." However, "it is also a mistake to accept observed test scores as either infallible or immutable" (p. 276).

In addition, the report helps to frame the dilemmas that arise from asking the same test to serve multiple functions and identifies seven distinct purposes of student assessment as a policy instrument (pp. 33–37):

- Aid in making instructional decisions about individual students.
- Provide information about the status of the educational system.
- Motivate change by "shaking people up."
- Evaluate programs.
- To hold schools and educators accountable for student performance.
- Leverage change in classroom instruction.
- To certify individual students as having attained specified levels of achievement.

The report seeks to clarify the relationship between the types and forms of assessment used and the purposes for which the assessment is given. It argues that standards-based approaches and accountability approaches can be compatible or incompatible, depending on what the tests measure, how they are used, and the regulations that govern their implementation and influence.

In making its recommendations, the report provides a clear picture of the tensions that abound on the assessment landscape. The reader is reminded of the tension between the enthusiasm of policymakers and the caution of experts that results in the twin dilemmas that (1) policy and public expectations of testing generally exceed the technical capacity of the tests themselves, and (2) the desire for more fairness and efficiency often conflicts with the impulse to sort and classify students (pp. 30–31).

The committee indicated a "strong need for better evidence on the intended benefits and unintended negative consequences of using high-stake tests to make decisions about individuals" (p. 8).

The report concludes that "large-scale assessments, used properly, can improve teaching, learning, and equality of educational opportunity" (p. 9). But, "when test use is inappropriate, especially in the case of high-stakes decisions about individuals, it can undermine the quality of education and equality of opportunity" (p. 276). Thus assessments have the potential for both help and harm, which should motivate action to ensure that educational tests are used fairly and effectively.

RECOMMENDATIONS MADE IN THE REPORT

The following recommendations represent a selection of the findings and recommendations presented in the report (pp. 275–290).

- Decisions regarding appropriate use of tests should be based on the following principles:

  – "First, the important thing about a test is not its validity in general, but its validity when used for a specific purpose.
  – Second, tests are not perfect. Test questions are a sample of possible questions that could be asked in a given area. Moreover, a test score is not an exact measure of a student's knowledge or skills.
  – Third, an educational decision that will have a major impact on a test taker should not solely or automatically be made on the basis of a single test score.
  – Finally, neither a test score nor any other kind of information can justify a bad decision." (p. 275)

- "Accountability for educational outcomes should be a shared responsibility of states, school districts, public officials, educators, parents, and students. High standards cannot be established and maintained merely by imposing them on students." (p. 278)
- "As tracking is currently practiced, low-track classes are typically characterized by an exclusive focus on basic skills, low expectations, and the least-qualified teachers. Students assigned to low-track classes are worse off than they would be in other placements. This form of tracking should be eliminated. Neither test scores nor other information should be used to place students in such classes." (p. 282)
- "Scores from large-scale assessment should never be the only sources of information used to make a promotion or retention decision. No single source of information—whether test scores, course grades, or teacher judgments—should stand alone in making promotion decisions. Test scores should always be used in combination with other sources of information about student achievement." (p. 286)
- "The quality of the process of setting a cutscore on a graduation test should be documented and evaluated—including the qualifications of the judges employed, the method or methods employed, and the degree of consensus reached." (p. 290)
- "Students who fail should have opportunities to retake any test used in making promotion decisions. This implies that tests used in making promotion decisions should have alternate forms." (p. 287)

- "All students are entitled to sufficient test preparation so their performance will not be adversely affected by unfamiliarity with item format or by ignorance of appropriate test-taking strategies." (p. 290)
- "In general, large-scale assessments should not be used to make high-stakes decisions about students who are less than 8 years old or enrolled below grade 3." (p. 279)

Recommendations related to assessing mathematical understanding can be found in *Principles and Standards* (p. 11), *Adding It Up* (p. 423–424), and *How People Learn* (p. 24).

ACTIONS EDUCATORS MIGHT CONSIDER

Based on the discussions, findings, and recommendations in *High Stakes,* educators and policymakers concerned with making critical decisions about tracking, promotion, and graduation might

- Examine all assessment policies and procedures currently in place to ensure that the spirit of fair and appropriate uses of student assessments permeates practice.
- Analyze the purposes for which any given student assessment was developed, and ensure that these intents match the actual uses of the assessment results.
- Ensure that no high-stakes decision about an individual student is ever made on the basis of a single measure.

6

What Must Change?

> In a practical sense, mathematics skills and understanding are of vital importance in determining the future success of today's young people. But unfortunately, their mathematics curriculum in too many cases does not prepare them for what the future will demand. …Change has not come far enough or fast enough to ensure that all of our 46 million public school students are afforded equal opportunity to learn everything they are capable of in school, to guarantee to the nation a well-skilled workforce, or to assure our continued economic standing in the world community, (p. 5, *Every Child Mathematically Proficient*)

Improving mathematics education means improving the mathematics programs offered to students. Schools, districts, and states should begin by reflecting on the kind of mathematical knowledge and skills that students can possess and by considering how to structure a program to enable students to learn this knowledge and develop these skills. Once stakeholders—teachers, administrators, and community members—agree to a mathematics program, the next step is to think about how to make the program a reality. Educators considering their mathematics programs, need to ask some critical questions:

- What is the nature of the mathematics program?
- Where can we begin to make improvements?
- How do we cultivate support—both among our staff and from the community at large—for changes in our mathematics program?
- How can we ensure that all students leave school mathematically proficient?

RESOURCE AVAILABLE

Every Child Mathematically Proficient: An Action Plan of the Learning First Alliance, November 1998.

OVERVIEW OF THE RESOURCE

Every Child Mathematically Proficient is an action plan that summarizes current conditions in mathematics education, sets forth a straightforward goal for improvement, and supports this goal with four objectives and a series of action steps. "Together," the report notes, "we have a major challenge to raise achievement throughout our nation. The Learning First Alliance, therefore, advances this Action Plan to bring American students to world-class levels in mathematics" (p. 2). The Learning First Alliance, formed in January 1997, is a collaboration of 12 national organizations[1] working to improve student learning in America's public elementary and secondary schools. None of these organizations are directly connected to mathematics education and, as a consequence, this document arises outside of the mathematics education community. The statements, however, are supportive of and consistent with the findings of other, more specific, reports that are summarized in *Improving Mathematics Education*. The document was developed through consultation with diverse cross-sections of the mathematics education community. It has served as a focal point for efforts by members of the Learning First Alliance in their efforts to improve mathematics education and others have used its recommendations, for example, the Conference Board of the Mathematical Sciences in developing *the Mathematical Education of Teachers*.

The proposed action plan is predicated on the following:

> Our goal is for virtually all students to successfully complete a challenging K–12 mathematics curriculum that includes mastery of the content included in the two one-year Algebra I and Geometry courses by the end of grade nine. (p. 7)

[1]American Association of Colleges for Teacher Education, American Association of School Administrators, American Federation of Teachers, Association for Supervision and Curriculum Development, Council of Chief State School Officers, Education Commission of the States, National Association of State Boards of Education, National Association of Elementary School Principals, National Association of Secondary School Principals, National School Boards Association, National Parent Teacher Association, National Education Association.

An endnote explains that middle school students must be well grounded in Number Sense, Properties and Operations; Measurement; Data Analysis, Statistics, and Probability; Algebra and Functions; and Geometry and Spatial Sense, but the Alliance is focusing on Algebra and Geometry because they are powerful gatekeepers for access to post-secondary education studies and key jobs. The endnote also states that the use of the word "course" does not imply a preference for the existing course structure (p. 29).

The plan acknowledges that meeting this goal requires curricular changes, professional development, parental and public support, and research-based reforms. These lead to four objectives for schools, school districts, and states (p. 6):

- "All our nation's students, regardless of where they live or their economic or racial and ethnic backgrounds, should have the opportunity to complete a challenging course of mathematics study that is consistent with specific benchmarks, including Algebra and Geometry by the end of the ninth grade.
- Students must be taught by teachers who have a strong command of the subject and the best ways to teach it, which will require changes in preservice teacher education, increased entry requirements for the initial education of teachers, and continued professional development of teachers throughout the full range of their careers.
- Parents and teachers must be brought into the process of change in school mathematics, including discussions of curricular goals, how teaching and assessments have changed in mathematics classrooms, and how they may help improve student achievement.
- Finally, programs of research on curricular materials, student learning, and teaching of school mathematics should be expanded. More support should be given to the translation of findings from such research into the development of high quality materials and professional development opportunities for teachers."

RECOMMENDATIONS MADE IN THE REPORT

To those seeking to achieve the four objectives, *Every Child Mathematically Proficient proposes* a series of practical recommendations, including the following:

- "At the state and school district level, specify clear benchmarks and provide a more focused and challenging study of mathematics for each grade or group of grades. Teacher preparation, textbooks and other curriculum materials, assessments, and mechanisms for holding schools accountable should be aligned with these benchmarks." (p. 14)

- "Eliminate dead-end tracks in the school curriculum, such as 'general mathematics.'" (p. 15)
- "Develop clear, consistent, and regularly administered assessment programs for monitoring student progress toward curriculum benchmarks." (p. 15)
- "Continue to study how technology should be used to further student learning in mathematics." (p. 15)
- "Bring all pre-service teacher education programs into line with the standards for what teachers should know about mathematics and mathematics education established by the Mathematical Association of America, Interstate New Teacher Assessment and Support Consortium and the National Council of Teachers of Mathematics." (p. 22)
- "Develop, support, and require teacher professional development in mathematics and mathematics education over the full span of teaching careers, with special emphasis on the first years of induction into the profession and on continued growth in teaching mathematics." (p. 22)
- "All students of mathematics should be taught by teachers who have been well prepared in the content of mathematics and techniques of teaching mathematics; in particular, all mathematics teachers in grades five through nine should be mathematics specialists." (p. 16)
- "Carefully evaluate the relative effectiveness of varied approaches to achieving standards for school mathematics for students, for teachers, and for instructional programs as a whole." (p. 27)
- "Continue to monitor national and international achievement and curricular trends to provide a basis for comparison and targets for improvement." (p. 28)
- "Equip teachers with tools and supports to enable them to help children of all backgrounds complete a challenging mathematics curriculum." (p. 22)
- "Translate research findings into strategies to improve the effectiveness of various instructional approaches, commercial and project materials, and the use of technology to foster student achievement and increase rates of student retention in school mathematics programs." (p. 28)

ACTIONS EDUCATORS MIGHT CONSIDER

The summaries of current practice and proposed actions laid out in *Every Child Mathematically Proficient* can be a starting point from which to initiate discussions of change and improvement of mathematics programs. To consider how to begin the process and to ensure that all students leave school mathematically proficient, educators and policy makers might do the following:

- Analyze their existing mathematics program in light of the recommendations in *Every Child Mathematically Proficient.*

- Work together to devise a research-based plan for changing and aligning curriculum, instruction, and assessments to raise standards and levels of achievement.
- Bring school board members, administrators, and teachers together to use the summaries of current levels of achievement, curriculum organization, and teaching practice found in Sections 3 and 4 of *Every Child Mathematically Proficient* to initiate discussions, stimulate consideration of changes, and frame locally appropriate initiatives.
- Review the recommended action steps to determine how to foster change and where the work of the Learning First Alliance might be of assistance.

7

Conclusion

The eight documents summarized in this report have been categorized as informing content, learning, teaching, assessment, and program changes. The Committee felt that organizing the recommendations in the same way would enable readers to understand the connections among the documents and illustrate common themes. Tables 7–1–5 highlight those recommendations that seem to cut across the eight publications. While there are diverse opinions in the mathematical community on how to improve mathematics education, the reports contain noticeable overlap in both the spirit and the specifics of the recommendations.

The tables do not contain all of the recommendations summarized in *Improving Mathematics Education,* let alone the entire set of recommendations across the eight documents. Empty cells in the table should not be construed as lack of support; the publication may actually address the particular issue but not make a direct recommendation. And, although some topics were common to several documents, this does not necessarily imply the recommendations were the same. Note that despite the increased interest in assessment and accountability throughout education, little attention seems to be paid to the topic across these publications.

The Committee identified several cross-cutting themes that emerged from the documents by considering the recommendations in each section and categorizing them across topics that emerged from the sets of recommendations. These common themes can provide a background to help frame the decisions we must make to improve school mathematics in the United States.

Change. The world is changing, society's expectations are changing, and our students are changing. It follows that the mathematics needed, its importance, and methods for teaching it are changing as well. As *Before It's Too Late* states, “It is abundantly clear from the evidence clearly at hand that we are not doing the job we should.” Five of the documents (see Table 7–5) reflect the need to change what we do, how we do it, and how for all students. For example, *Adding It Up* makes direct statements about

Table 7-1 What Should We Teach?

Recommendations in key categories	Principles and Standards	Adding It Up	How People Learn	Math Education of Teachers	Educating Teachers	Before It's Too Late	High Stakes	Every Child
The documents that:								
Emphasize problem solving	x	x	x	x				
Emphasize mathematical reasoning	x	x	x	x				x
Suggest the use of technology in the development of mathematical proficiency	x	x	x			x		x
Advocate challenging mathematics for all students	x							x
Advocate algebra and geometry early in the curriculum	x	x						x
Emphasize mathematics as sense making	x	x		x				

Table 7-2 How Should We Teach?

Recommendations in key categories	Principles and Standards	Adding It Up	How People Learn	Math Education of Teachers	Educating Teachers	Before It's Too Late	High Stakes	Every Child
The documents that:								
Advocate that teachers build on what students know and on student thinking	x	x	x		x			
Emphasize the importance of understanding how to teach	x	x	x	x				x
Emphasize teaching so students learn with understanding	x	x	x					
Support instruction that integrates skills and conceptual knowledge	x	x						
Emphasize the importance of having knowledge taught in a variety of contexts			x	x				

Table 7-3 What Preparation and Support Do Teachers Need?

Recommendations in key categories	Principles and Standards	Adding It Up	How People Learn	Math Education of Teachers	Educating Teachers	Before It's Too Late	High Stakes	Every Child
The documents that:								
Advocate ongoing professional development		x	x	x	x	x		x
Support programs for novice teachers					x	x		x
Emphasize the need for strong content knowledge on the part of teachers	x	x	x	x	x			x
Promote K–12 partnerships to improve teacher education				x	x	x		
Suggest that postsecondary educators assume more responsibility for preparing and developing teachers				x	x			

Table 7-4 How Do We Know That What We Are Doing Is Working?

Recommendations in key categories	Principles and Standards	Adding It Up	How People Learn	The Math Education of Teachers	Educating Teachers	Before It's Too Late	High Stakes	Every Child
The documents that:								
Advocate using multiple ways to assess student understanding	x						x	
Emphasize the use of assessment to support the learning of mathematics	x	x	x					
Caution about the implications of high-stakes assessments for instruction	x							

Table 7-5 What Must Change?

Recommendations in key categories	Principles and Standards	Adding It Up	How People Learn	Math Education of Teachers	Educating Teachers	Before It's Too Late	High Stakes	Every Child
The documents that:								
Emphasize the need to align curriculum, instruction, and assessment	x	x			x	x		x
Advocate expanding and using research to improve mathematics education		x	x		x		x	x
Support the use of math specialists or teacher leaders	x	x		x				x
Emphasize high expectations for all students	x	x				x	x	x
Advocate engaging parents in improving mathematics education	x	x						x
Call for system changes		x		x	x	x		x
Stress coherent, important mathematics that is well articulated throughout the grades	x	x						x

we assess the impact of the changes to provide better instructional programs problems in the current system: "Our experiences, discussions, and review of the literature have convinced us the school mathematics demands substantial change" (p. 407). It also suggests coordination of curriculum, instructional materials, assessment, and professional development; school organization should drive the changes. *Every Child Mathematically Proficient* offers an action plan for changing mathematics education. *The Mathematical Education of Teachers* calls for "changing expectations for mathematical knowledge."

Equity. According to the documents, all students can and should be mathematically proficient. The reports consistently deliver the message that school mathematics can and must empower all students (see Table 7–5). For example, five of the documents emphasize high expectations for all students. *Every Child Mathematically Proficient* calls for ending "dead-end" tracks, and *High Stakes* calls for the end of tracking that discriminates

against certain students. The content areas emphasized in the documents (see Table 7–1) underscore that "mathematics for all" does not mean a watered-down mathematics but an authentic, coherent, and functional mathematics that expects more of every student than in the past. For example, *Every Child Mathematically Proficient* recommends algebra and geometry for all students, and a majority of the documents emphasize mathematical reasoning.

Content. Mathematical content is the centerpiece for teaching, instructional materials, assessments, teacher preparation, and professional development. The documents make strong statements about the importance of reasoning, problem solving, algebra, and geometry (Table 7–1). For example, reasoning and proof is a content strand in *Principles and Standards for School Mathematics,* and *Adding It Up* includes the notion of adaptive reasoning as essential for mathematical proficiency. Six of the documents call for strong content knowledge on the part of teachers (Table 7–3) and emphasize the need to align curriculum instruction and assessment (Table 7–5). In particular, *Before It's Too Late* states "high-quality teaching requires that teachers have a deep knowledge of subject matter" (p. 22), while *The Mathematics Education of Teachers* suggests the specific number of content hours that should be part of undergraduate teacher preparation programs. If teachers do not fully understand fundamental mathematics, it is unlikely they will impart that knowledge to their students.

Understanding. While research findings in education are always open to further interpretation, it is increasingly clear that learning is a complex cognitive process that builds on prior knowledge and requires active engagement with new situations. As described in *How People Learn,* a growing body of knowledge suggests that learning is best accomplished when accompanied by conceptual knowledge and making sense of new concepts, with an emphasis on understanding. *Adding It Up* makes this explicit in terms of learning mathematics, emphasizing the need for both conceptual understanding and procedural fluency. Four of the documents (Table 7–2) support the use of student thinking to build understanding. The notion of learning as a sense-making process also has import for teaching and teacher preparation with recommendations that teachers be prepared to teach mathematics in ways that enable students to learn. For example, *Educating Teachers of Science, Mathematics, and Technology* suggests that teachers "regularly update the content knowledge and pedagogical tools needed to teach in ways that enhance student learning and achievement in these subjects" (p. 109).

Research. The use of research to support mathematics education is a key element in majority of the documents (Table 7–5). This ranges from *How People Learn,* which calls on existing research to substantiate claims made in the document, to *High Stakes,* which advocates advanced research on

topics for which there are more questions than answers. *Educating Teachers of Science, Mathematics, and Technology* calls for enhanced links between research and practice, and *Every Child Mathematically Proficient* suggests such links should be translated into the development of curricular materials and professional development programs for teachers. The prominence of research in these documents suggests a shift in the way changes are made in mathematics education and has implications for how program changes are instituted.

Collaboration. Making change in our complex education system is difficult, and improvement requires collaboration among diverse stakeholders who represent different parts of the system. Systemic change requires new forms of partnerships to make the system more productive and to provide solutions that cut across the system components. *Educating Teachers of Science, Mathematics, and Technology* and *The Mathematics Education of Teachers* make strong statements about forming K–12 and university partnerships, involving mathematicians and scientists to improve teacher preparation and development (Table 7–3). Several of the documents suggest that parents as well as teachers must be brought into the process of change in school mathematics, including discussions of curricular goals and how teaching and assessments have changed in mathematics classrooms (Table 7–5).

Professional Development. Every report that addresses changing conditions and expectations eventually arrives at the need for more effective forms of professional development. Six of the reports (Table 7–3) support the need for highly qualified teachers who take part in continuous learning that is part of the system for teachers: "High-quality teaching must be continually reshaped" (*Before It's Too Late,* p. 22). Given enhanced opportunities for professional growth and development, teachers will be able to implement the vision of mathematics instruction called for in these reports. Moreover, the professional development must be more than a "patchwork of courses" (*Educating Teachers,* p. 31) and must be related to the mathematics teachers are expected to teach and to how to helping students understand this mathematics.

Technology. The documents recognize technology as a major factor driving the need for change, creating a different world with different expectations about the mathematics students will need (Table 7–1). The reports, however, are not unanimous in their technology recommendations. *Principles and Standards* states that technology is essential to the teaching and learning of mathematics. *Every Child Mathematically Proficient* cautions that further study is needed to explore how technology should be used to enhance student learning in mathematics, and *Adding It Up* is also cautious, but suggests that technology offers much promise in the search for ways to improve what happens in classrooms.

Mathematics Specialists. As teachers learn new content, new methods of instruction, and new methods of assessment, they need resources and support. There is also immense pressure on elementary teachers, who have multiple subjects to teach. While math specialists are called for in several of the documents (Table 7–5), the nature of such specialists and the recommendations vary. *The Mathematics Education of Teachers* and *Every Child Mathematically Proficient* call for math specialists to teach mathematics content by grade 5. *Adding It Up* recommends that mathematics specialists provide support for K–8 teachers, and *Principles and Standards* calls for teacher-leaders throughout the system.

Closing Comments

Considering the overall mathematical landscape and potential directions necessary for improvement, the Mathematical Sciences Education Board recommends that educators use reports such as the ones described in this document to help them make informed decisions. The committee also recommends that the decision making process include participation and input from across the community, with diverse contributions from a wide spectrum of individuals—in much the same spirit as these documents were developed.

The committee recognizes that all the information necessary to make the many decisions facing educators is not available. The research-based findings and the recommendations in these documents, however, can be a rich resource for educators engaged in actions and initiatives intended to make real and positive change in the lives of students and teachers in mathematics classrooms across the nation. These documents can help educators as they struggle to make wise and defensible decisions and strive to meet the challenges of implementing truly excellent programs of mathematics instruction.

Securing Copies of the Resources Summarized

Adding It Up: Helping Children Learn Mathematics is available from the National Academy Press, 2101 Constitution Avenue, NW, Washington, DC 20055, or online at www.nap.edu.

Before It's Too Late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21st Century is available from the U.S. Department of Education, Education Publications Center, PL Box 1398, Jessup, MD 20794, or online at www.ed.gov/americacounts/glenn.

Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium is available from the National Academy Press, 2101 Constitution Avenue, NW, Washington, DC 20055, or online at www.nap.edu.

Every Child Mathematically Proficient: An Action Plan of the Learning First Alliance is available from the Learning First Alliance, 1001 Connecticut Avenue, Suite 335, Washington, DC 20036, or online at www.learningfirst.org/mathaction.html.

High Stakes: Testing for Tracking, Promotion, and Graduation is available from the National Academy Press, 2101 Constitution Avenue, NW, Washington, DC 20055, or online at www.nap.edu.

How People Learn: Brain, Mind, Experience, and School—Expanded Edition is available from the National Academy Press, 2101 Constitution Avenue, NW, Washington, DC 20055, or online at www.nap.edu.

The Mathematical Education of Teachers is available from the Conference Board of the Mathematical Sciences at 1529 Eighteenth Street, NW, Washington, DC 20036, or online at http://www.maa.org/cbms/MET_Document/index.htm.

Principles and Standards for School Mathematics is available from the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191, or online at www.nctm.org/standards.

References

Conference Board of the Mathematical Sciences. (2001). *The Mathematical Education of Teachers* . Washington, DC: Author.

Learning First Alliance. (1998). *Every Child Mathematically Proficient: An Action Plan of the Learning First Alliance* . Washington, DC: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics* . Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics* . Reston, VA: Author.

National Research Council. (1999). *High Stakes: Testing for Tracking, Promotion, and Graduation* . Committee on Appropriate Test Use. J. Heubert and R.Hauser (Eds.). Board on Testing and Assessment, Divisionof Behavioral and Social Sciences and Education. Washington, DC:National Academy Press.

National Research Council. (2000). *Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium* . Committee on Science and Mathematics Teacher Preparation. Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council. (2000). *How People Learn: Brain, Mind, Experience, and School: Expanded Edition* . Committee on Developments in the Science of Learning and the Committee on Learning Research and Educational Practice. J.Bransford, A.Brown, R.Cocking, S.Donovan, and J.Pellegrino (Eds.). Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council. (2001). *Adding It Up: Helping Children Learn Mathematics* . Mathematics Learning Study Committee. J.Kilpatrick, J. Swafford, and B.Findell (Eds.). Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council. (2001). *Knowing What Students Know: The Science and Design of Educational Assessment* . Committee on the Foundations of Assessment. J.Pellegrino, N.Chudowsky, and R.Glaser(Eds.). Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

U.S. Department of Education. (2000). *Before It's Too Late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21 st Century* . Washington, DC: Author.