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Concise Hydrology

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Content

Preface

Hydrology is a branch of scientific and engineering discipline that deals with the occurrence, distribution, movement, and properties of the waters of the earth. A knowledge of hydrology is fundamental to water and environmental professionals (engineers, scientists and decision makers) in such tasks as the design and operation of water resources, wastewater treatment, irrigation, flood defence, navigation, pollution control, hydropower, ecosystem modelling, etc. This is an introductory book on hydrology and written for undergraduate students in civil and environmental engineering, environmental science and geography. The aim of this book is to provide a concise coverage of key contents in hydrology that is easy to access through the Internet.

The book covers the fundamental theories on hydrological cycle (water balance, atmospheric water, subsurface water, surface water), precipitation analysis, evaporation and evapotranspiration processes, infiltration, ground water movement, hydrograph analysis, rainfall runoff modelling (unit hydrograph), hydrological flow routing, measurements and data collection, hydrological statistics and hydrological design. The text has been written in a concise format that is integrated with the relevant graphics. There are many examples to further explain the theories introduced. The questions at the end of each chapter are accompanied by the corresponding answers and full solutions. A list of recommended reading resources is provided in the appendix for readers to further explore the interested hydrological topics.

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1. Introduction

Hydrology is a branch of scientific and engineering discipline that deals with the occurrence, distribution, movement, and properties of the waters of the earth. Knowledge of hydrology is fundamental to water and environmental professionals (engineers, scientists and decision makers) in such tasks as the design and operation of water resources, wastewater treatment, irrigation, flood risk management, navigation, pollution control, hydropower, ecosystem modelling, etc.

This unit covers the fundamental theories on 1. Hydrological cycle and water balance, 2. Precipitation, 3. Evaporation and transpiration, 4. Infiltration, 5. Groundwater, 6. Hydrograph, 7. Flow routing, 8. Hydrological measurements, 9. Hydrological statistics, 10. Hydrological design.

1.1 Hydrological Cycle

The water cycle, also known as the hydrologic cycle, describes the continuous movement of water on, above and below the earth surface. The sun, which drives the water cycle, radiates solar energy on the oceans and land. Water evaporates as vapor into the air. Ice and snow can sublimate directly into water vapor. Evapotranspiration is water transpired from plants and evaporated from the soil. Rising air currents take the vapor up into the atmosphere where cooler temperatures cause it to condense into clouds. Air currents move clouds around the globe, cloud particles collide, grow, and fall out of the sky as precipitation. Some precipitation falls as snow and can accumulate as ice caps and glaciers, which can store frozen water for thousands of years. Snowpacks can thaw and melt, and the melted water flows over land as snowmelt. Most precipitation falls back into the oceans or onto land, where the precipitation flows over the ground as surface runoff. A portion of runoff enters rivers in valleys in the landscape, with streamflow moving water towards the oceans. Runoff and groundwater are stored as freshwater in lakes. Not all runoff flows into rivers. Much of it soaks into the ground as infiltration. Some water infiltrates deep into the ground and replenishes aquifers, which store huge amounts of freshwater for long periods of time. Some infiltration stays close to the land surface and can seep back into surface-water bodies (and the ocean) as groundwater discharge. Some groundwater finds openings in the land surface and comes out as freshwater springs. Over time, the water returns to the ocean, where our main water cycle started (Wikipedia, 2009).

1.2 Key Hydrological Processes

Precipitation : Condensed water vapor that falls to the earth surface. Most precipitation occurs as rain, but also includes snow, hail, fog drip, sleet, etc.

Runoff: The variety of ways by which water moves across the land. This includes both surface runoff and channel runoff. As it flows, the water may infiltrate into the ground, evaporate into the air, become stored in lakes or reservoirs, or be extracted for agricultural or other human uses.

Infiltration: The flow of water from the ground surface into the ground. Once infiltrated, the water becomes soil moisture or groundwater.

Subsurface Flow: The flow of water underground, in the vadose zone and aquifers. Subsurface water may return to the surface (e.g. as a spring or by being pumped) or eventually seep into the oceans. Water returns to the land surface at lower elevation than where it infiltrated, under the force of gravity or gravity induced pressures. Groundwater tends to move slowly, and is replenished slowly, so it can remain in aquifers for thousands of years.

Evaporation and transpiration: The transformation of water from liquid to gas phases as it moves from the ground or bodies of water into the overlying atmosphere. The source of energy for evaporation is primarily solar radiation. Evaporation often implicitly includes transpiration from plants, though together they are specifically referred to as evapotranspiration.

1.3 Common Units

Flow rate in stream and rivers are usually recorded as cubic metres per second (m^3/s, i.e., cumecs) or cubic feet per second (cfs). Volumes are often measured as cubic metres, gallons, and litres. Precipitations are commonly recorded in inches or millimetres. Rainfall rates are usually represented in inches or centimetres per hour. Evaporation, transpiration and infiltration rate are measured as inches or millimetres per day or longer time periods.

Some common conversions:

1 inch = 0.254 metre = 25.4 mm
1 foot = 0.3048 metre
1 gallon = 0.003785 m^3
1 m^3 = 1000 litres
1 mile = 1.609 km

1.4 Water Distribution in Space and Time

The estimates of the total amount of water on the earth and in various processes are presented in Table 1. It can be seen that most of the earth's water is in the oceans (96.5%). Fresh water is only a small proportion of the total water (2.5%) and mainly stored in the ice.

Table 1 Inventory of world water quantities (Chow, et al., 1988)

Reservoir	Volume (cubic km x 1,000)	Percent of total	Percent of fresh water
Oceans	1338000	96.5	
Ice Caps and Glaciers	24364.1	1.8	69.6
Groundwater (Fresh)	10530	0.76	30.1
Groundwater (Saline)	12870	0.93	
Lakes (Fresh)	91	0.007	0.3
Lakes (Saline)	85.4	0.006	
Soil Moisture	16.5	0.001	0.05
Atmosphere	12.9	0.001	0.04
Streams and Rivers	2.12	0.000	0.006
Marshes	11.47	0.001	0.03
Biosphere	1.12	0.000	0.003
Total	1385985	100.0	
Fresh water	35029	2.5	100

The residence time is the average duration for a water molecule to pass through a water body. It can be derived by dividing the volume of water by the flow rate. Some estimated residence time values are listed in Table 2.

Table 2 Average residence time (Wikipedia, 2009)

Water body	Average residence time
Oceans	2600 to 3200 years
Glaciers	20 to 100 years
Seasonal snow cover	2 to 6 months
Soil moisture	1 to 2 months
Groundwater: shallow	100 to 200 years
Groundwater: deep	10,000 years
Lakes	50 to 100 years
Rivers	2 to 6 months
Atmosphere	days

1.5 Water Balance

The total amount of water available to the earth is finite and conserved. Although the total volume of water in the global hydrologic cycle remains constant, the distribution of this water is continually changing on continents, in regions and local catchments. The global annual water balance is presented in Table 3.

Table 3 Global annual water balance (Chow, et al., 1988)

	Ocean	Land
Area (x10^6km^2)	361.3	148.8
Precipitation (x10^3 km^3/yr)	458	119
Precipitation (mm/yr)	1270	800
Evaporation (x10^3 km^3/yr)	505	72
Evaporation (mm/yr)	1400	484
Runoff to ocean		
Rivers (x10^3 km^3/yr)		44.7
groundwater (x10^3 km^3/yr)		2.2

From the conservation of mass, water balance for any storage can be expressed as

$$Q_I - Q_O = \frac{dS}{dt} \tag{1}$$

where Q_I, Q_O - input flow rate, output flow rate; S - storage

For a discrete system with a time duration Δt, Eq(1) can be expressed as

$$V_I - V_o = \Delta S \tag{2}$$

where V_I and V_o are input volume and output volume; ΔS is storage change

1.6 Catchment

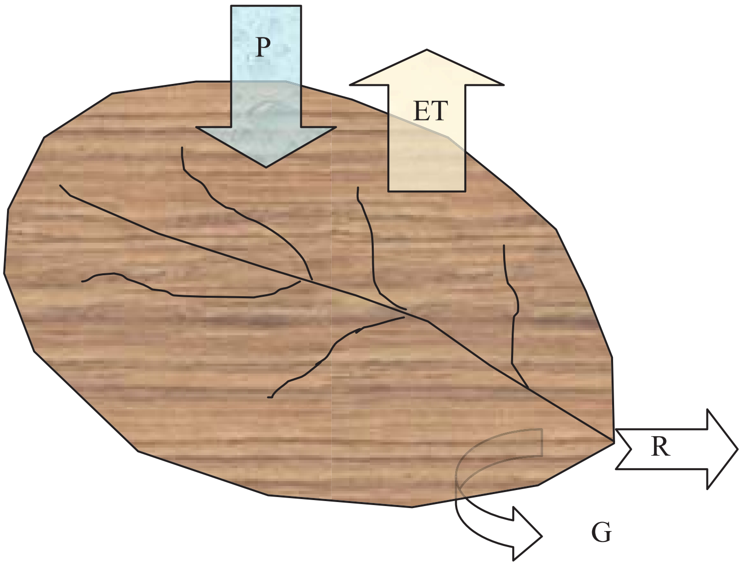


Figure 1 Catchment water balance

A catchment (also called drainage basin, river basin, watershed) is an extent of land where water from rain or snow melt drains downhill into a body of water, such as a river, lake, reservoir, estuary, wetland, sea or ocean. In hydrology, catchment is a logical unit of focus for studying the movement of water within the hydrological cycle, because the majority of water that discharges from the catchment outlet originated as precipitation falling on the catchment.

The water balance equation for a catchment

$$P - R - G - ET = \Delta S \tag{3}$$

where P - precipitation, R - river runoff, G - groundwater runoff, ET - evapotranspiration, ΔS - storage change in a catchment.

1.7 Practice

Practice 1
The volume of atmospheric water is 12,900 km^3. The evapotranspiration from land is 72,000km^3/year and that from ocean is 505,000km^3/year. Estimate the residence time of water molecules in the atmosphere (in days).

Solution

The residence time can be derived by dividing the volume of water by the flow rate

Total flow rate = 505000+72000=577000 km^3/s

The residence time = 12900/577000 = 0.0224 year = 8.2 days

Practice 2

A reservoir has the following inflows and outflows (in cubic meters) for the first three months of the year. If the storage at the beginning of January is 60m^3, determine the storage at the end of March.

Month	Jan	Feb	Mar
Inflow	4	6	9
Outflow	8	11	5

Solution

The storage change is

$\Delta S = I - O = (4+6+9)-(8+11+5) = -5m^3$

The storage is 60-5=55m^3

Questions 1
Introduction

1. Describe the hydrological cycle and its key processes.

2. The total amount of water in the atmosphere is 12.9×10^3 km^3. Estimate the depth of precipitation if the atmosphere water is completely transformed to precipitation (treat the earth as a sphere with a mean radius of 6,371km and the sphere surface area equation is $4\pi R^2$).

   (Answer: 25 mm)

3. About 577,000 km^3 of water fall as precipitation each year on the earth, calculate the average annual depth of precipitation on the earth surface (in millimetres).

   (Answer: 1131 mm)

4. The volume of ocean water is 1338×10^6 km^3. The runoff from rivers is 44.7×10^3 km^3/year and the runoff from groundwater is 2.2×10^3 km^3/year. The precipitation on the ocean is 1270 mm /year (The ocean area is 361.3×10^6 km^2). Estimate the residence time of water molecules in the ocean (in years).

   (Answer: 2646 years)

5. The average annual precipitation in England and Wales is 926.9mm. A person consumes 150 litres of water every day (include agriculture, industry, trade, ..). With a population of 53,390,300 in England and Wales and an area of 58,368 square miles, what percentage of the precipitation is used by humans?

(Answer: 2.1%)

6. In a given year, a catchment with an area of 2500km^2 received 130 cm of precipitation. The average flow rate measured in the river draining the catchment was 30m^3/s.

   1) How much runoff reached the river for the year (in m^3)?
   2) Estimate the amount of water lost due to the combined effects of evapotranspiration and infiltration to groundwater (in m^3)?
   3) How much precipitation is converted into river runoff (in percentage)?

(Answers: $946\times10^6 m^3$, $2.3\times10^9 m^3$, 29%*)*

Solutions 1

Introduction

2. The earth surface area is $A = 4\pi R^2 = 4\pi \times 6371^2 = 510 \times 10^6 km^2$

   The average precipitation depth is
   $12.9 \times 10^3 \times 10^9 / (510 \times 10^6 \times 10^6) = 0.025m = 25mm$

3. The earth surface area is $A = 510 \times 10^6 km^2$

   The average annual precipitation depth is
   $577 \times 10^3 \times 10^9 / (510 \times 10^6 \times 10^6) = 1.131m = 1131mm$

4. The residence time can be derived by dividing the volume of water by the flow rate

   The flow rate is
   $44.7 \times 10^3 + 2.2 \times 10^3 + 1270 / 1000 / 1000 \times 361.3 \times 10^6 = 505751 km^3 / year$

   The residence time is $1338 \times 10^6 / 505751 = 2646 years$

5. The average annual precipitation in England and Wales is 926.9mm. A person consumes 150 litres of water every day (include agriculture, industry, trade,). With a population of 53,390,300 in the region and an area of 58,368 square miles, what percentage of the precipitation is used by humans?
   (Answer: 2646 years)

   The total consumption of water by humans in a year

   population x 150/1000 x 365 = $2.9 \times 10^9 m^3$

   The total area in m^2 58368 x 1609 x 1609 = $1.51 \times 10^{11} m^2$

   The water consumed by the total population in metres
   $2.9 \times 10^9 / (1.51 \times 10^{11}) = 0.0192m = 19.2mm$

   The percentage of the precipitation consumed is $19.2 / 926.9 = 0.0207 = 2.1\%$

6. 1) The total runoff volume is $30 \times 3600 \times 24 \times 365 = 946 \times 10^6 m^3$

2) The total precipitation is
$130/100 \times 2500 \times 10^6 = 3.25 \times 10^9 m^3$
Hence the loss is $3.25 \times 10^9 - 946 \times 10^6 = 2.3 \times 10^9 m^3$

3) The percentage precipitation converted into river runoff is
$946 \times 10^6 /(3.25 \times 10^9) = 0.29 = 29\%$

2. Precipitation

Precipitation is part of the atmosphere water and derived from water vapour. Atmospheric water mostly exists as vapour, but briefly and locally it becomes a liquid (rainfall and cloud water droplets) or a solid (snowfall, cloud ice crystal and hails).

2.1 Atmosphere Water

The sun is the driving force for the hydrological cycle. Precipitation comes from water vapour generated by the solar radiation from land and ocean. Water is made of H_2O hence water vapour is lighter than air (low air pressure is linked with high moisture, hence more likely to rain). The energy required to vaporise water is $2.5\times10^6\,J/kg$ (specific latent heat for water vaporisation).

Practice 1
A storm with 100mm depth fell over an area of 100 km^2 within 2 hours. Estimate the energy and power release from this storm (in Joule and MW).

Solution
Volume/mass of water = $100/1000\times100\times10^6 = 10\times10^6\,m^3 = 10\times10^9\,kg$

Total energy = $10\times10^9\,kg\times2.5\times10^6\,J/kg = 25\times10^{15}\,J$

Power = energy/duration= $25\times10^{15}\,J/(2\times3600)s = 3.5\times10^{12}W = 3.5\times10^6\,MW$

2.2 Precipitation Types

Precipitation is derived from atmospheric water. Atmospheric moisture is a necessary but not sufficient condition for precipitation. Other factors such as wind, temperature, atmospheric pressure and local landscape can influence precipitation. Two processes can produce precipitation: ice crystal process (aerosols act as freezing nuclei. Ice crystals grow in size and fall to ground. They tend to melt before hitting the ground surface) and coalescence process (small cloud droplets increase their sizes due to contact with other droplets through collision).

Vertical transport of air masses is a requirement for precipitation. There are three major categories of precipitation:

1. Convective precipitation: Heated air near the ground expands and absorbs more water moisture. The warm moisture-laden air moves up and gets condensed due to lower temperature, thus producing precipitation. Convective precipitation spans from light shows to thunderstorms

with extremely high intensity.

2. Orographic precipitation: The uplifting of air is caused by natural barriers such as mountain ranges.

3. Cyclonic precipitation: The uneven heating of the earth's surface by the sun results high and low pressure regions, and air masses move from high pressure regions to low pressure regions. If warm air replaces colder air, the front is called a warm front. If cold air displaces warm air, its front is called a cold front.

2.3 Rain drop size and velocity

Rain drops may be considered as falling bodies that subject to gravitational, buoyancy and air resistance effects. Rain drop velocity at equilibrium (terminal velocity) is related to the square of rain drop diameter. Larger drops fall faster and are able to collect more water during the fall. However, if a drop is too large (about 6~7 mm in diameter), it tends to break into smaller droplets.

The force balance for a rain drop is $F_d = F_g - F_b$ (drag force = gravity force - buoyancy)
From fluid mechanics

$$C_d \rho_a D^2 \left(\frac{\pi}{4}\right)\frac{V^2}{2} = \rho_w g \left(\frac{\pi}{6}\right) D^3 - \rho_a g \left(\frac{\pi}{6}\right) D^3 \tag{1}$$

where ρ_w and ρ_a are the density of water and air (assumed as 1000kg/m^3 and 1.2kg/m^3 at sea level). C_d is drag coefficient (Table 1).

Table 1 Drag Coefficient (Chow, et al. 1988)

D	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0	5.0
Cd	4.2	1.66	1.07	0.815	0.671	0.517	0.503	0.559	0.66

The drop terminal velocity can be derived as

$$V = \sqrt{\frac{4gD}{3C_d}\left(\frac{\rho_w}{\rho_a} - 1\right)} \tag{2}$$

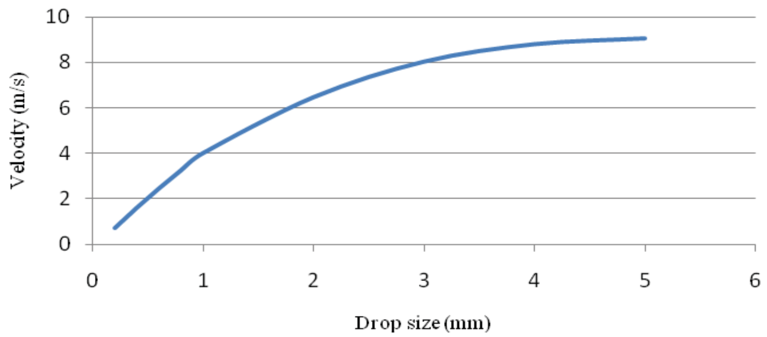


Figure 1 Typical rain drop velocity at the sea level

2.4 Precipitation data

Precipitation events are recorded by gauges at specific locations. Point precipitation data are used collectively to estimate areal variability of rain and snow. Rainfall data are usually represented as mm/hour, mm/day, etc.

Figure 2 A typical rain gauge in the UK

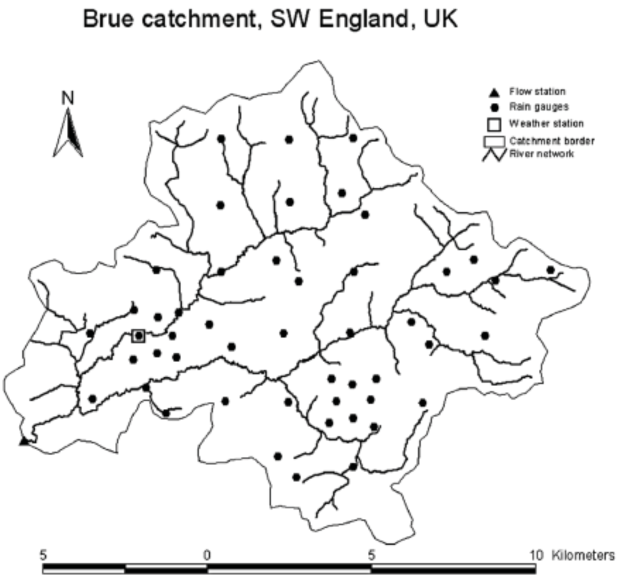


Figure 3 A rain gauge network at the Brue Catchment, UK

2.5 Double Mass Curve

A double mass curve is usually used to check the data quality of a specific rain gauge. A scatter plot is drawn between the interested gauge and a number of surrounding gauges.

Table 2 Rainfall records for Gauge X and other 20 gauges

Year	Gauge X	20 gauge average	Year	Gauge X	20 gauge average
2002	188	264	1984	223	360
2001	185	228	1983	173	234
2000	310	386	1982	282	333
1999	295	297	1981	218	236
1998	208	284	1980	246	251
1997	287	350	1979	284	284
1996	183	236	1978	493	361
1995	304	371	1977	320	282
1994	228	234	1976	274	252
1993	216	290	1975	322	274
1992	224	282	1974	437	302
1991	203	246	1973	389	350
1990	284	264	1972	305	228
1989	295	332	1971	320	312
1988	206	231	1970	328	284
1987	269	234	1969	308	315
1986	214	231	1968	302	280
1985	284	312	1967	414	343

The tasks involved are a) to examine the consistency of Gauge X data; b) to find when a change in regime occurred; c) to discuss possible causes; d) to adjust the data and determine what difference this makes to the 36 year annual average precipitation at Gauge X.

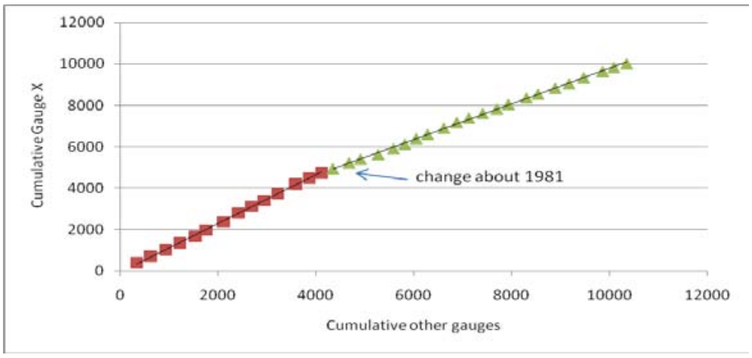


Figure 4 Double mass curve

It can be seen that Gauge X data are not consistent. There is a change in regime around 1981. This change could be due to gauge re-siting, growing trees, etc. If the earlier period is correct, the ratio of Gauge X to other gauges (1967-1981) is $\frac{\text{Gauge X average}}{\text{Other Gauge Average}} = \frac{330.7}{290.3} = 1.139$,

The ratio in the 2nd part (1982 - 2002) is $\frac{\text{Gauge X average}}{\text{Other Gauge Average}} = \frac{241.0}{285.67} = 0.8436$

Hence, the correction ratio should be $\frac{1.139}{0.8436} = 1.35$

All the rainfall values from 1982 to 2002 are applied with the same correction ratio (1.35).

The old average of Gauge X is 278.4 mm and the corrected one is 327.6mm

2.6 Areal Rainfall

It is important to have accurate rainfall information in a catchment for hydrological assessment. However, rainfall varies in space and it is expensive to install and maintain a very dense rain gauge network to completely cover all the catchments. As a result, only a limited number of gauges are installed and there are large gaps between the gauges. For assessing rainfall in a catchment, we need to determine the average rainfall over the catchment so that the total amount of rainfall could be estimated.

2.6.1 Arithmetic Mean

This is a simple method and can be used when the gauges are uniformly distributed.

$$\overline{R} = \frac{1}{n}\sum_{i=1}^{n} R_i \qquad (3)$$

2.6.2 Thiessen Polygon Method

The Thiessen Polygon method assumes that at any point in a catchment, the rainfall is the same as that at the nearest rain gauge so the depth recorded at a given gauge is applied out to a distance halfway to the next gauge in any direction.

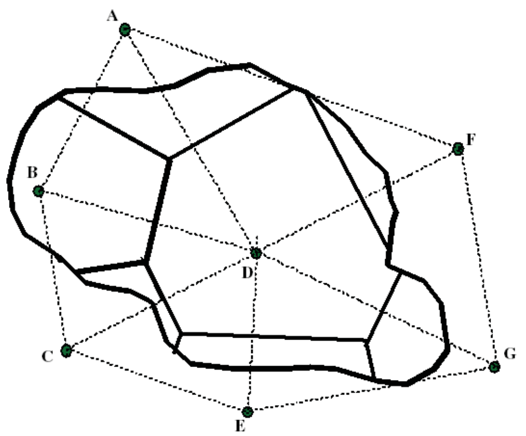


Figure 5 Thiessen Polygons

The relative weight for each gauge is determined from the corresponding area. If the area within the catchment assigned to each gauge is A_i, and its rainfall is Ri, the areal average rainfall for the catchment is

$$\overline{R} = \frac{1}{A}\sum_{i=1}^{n} A_i R_i \qquad (4)$$

where A is the total catchment area.

The Thiessen Polygon method is the most popular method used in practical engineering problems. The polygons can be plotted by hand or with computer software (such as ARCView and Matlab). However, it does not consider the gradual change between the gauges and ignores the orographic influence on rainfall.

Practice 2

Draw Thiessen polygons on the catchment shown in Figure 6. If the rainfall depths recorded by Gauge A, B and C are 10mm, 8mm and 9mm and the corresponding polygon areas are 5.1km^2, 3.2km^2 and 5.3km^2, estimate the catchment average rainfall depth.

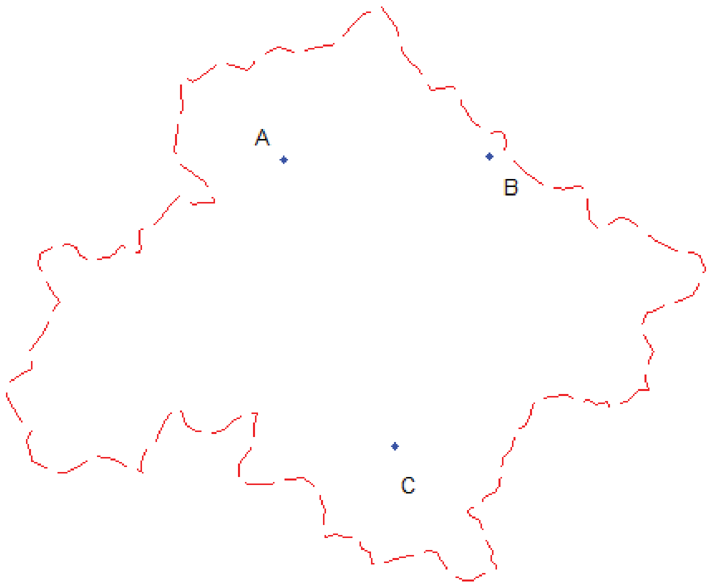
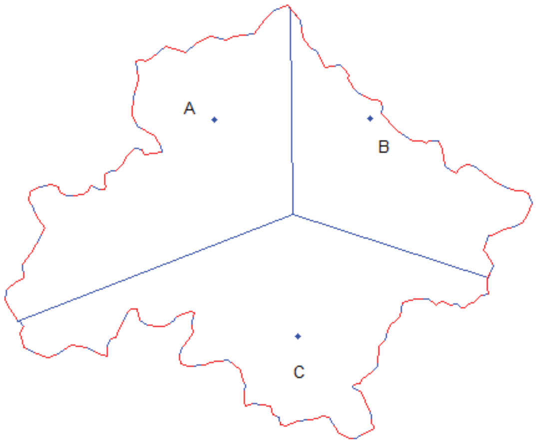


Figure 6 A catchment with three rain gauges

Solution



2.6.3 Isohyetal Method

This method uses isohyets constructed from the rain gauges by interpolating contour lines between adjacent gauges. Once the isohyetal map is constructed, the area between each pair of isohyets, within the catchment, is multiplied by the average rainfall depths of the two boundary isohyets. The average rainfall over the whole catchment can be estimated from the weight-averaged value.

The isohyetal method is flexible and knowledge of the storm pattern can help the drawing of isohyets, but a fairly dense network of gauges is needed to correctly construct the isohyetal map from a complex storm. They are useful for graphical display of rainfall distribution but less popular in engineering applications.

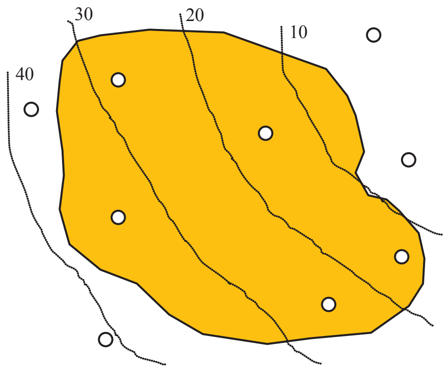


Figure 7 Isohyetal lines

2.6.4 Geostatistics

The conventional methods cannot estimate the uncertainty with the result. Geostatistical methods can be used to compute best estimates as well as error bands that describe the potential magnitude of the estimation error. The uncertainty information is useful for decision making (e.g., to add extra rain gauges if the uncertainties are large at certain points). Kriging is a typical method in this category. Readers can explore this method further at Wikipedia 'Kriging'.

Questions 2
Precipitation

1. The total amount of water in the atmosphere is 12.9x103 km^3. Calculate the energy release if the atmosphere water vapour is completely transformed to rainfall. How long time is needed if the total solar radiation on the earth surface is used to replenish the water vapour (the annual solar energy on the earth is 3.85 x10^{24} J)?

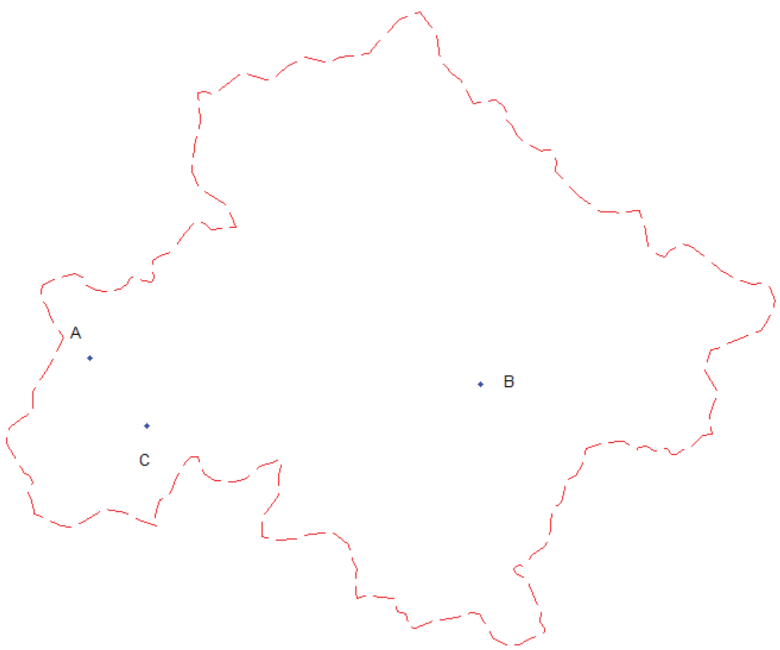
(Answer: $32\times10^{21} J$, 2.9 days)

2. What is the terminal velocity for a light rain with a drop size of 0.6 mm at sea level (Cd= 1.07, $\rho_a = 1.2 kg/m^3$, $\rho_w = 1000 kg/m^3$)? If the air density drops by 50% at 5km in the sky, will the same rain drop falls faster or slower? Calculate its velocity at this height (assume little change with g ρ_w and Cd). If a weather radar beam detects such a rain drop at 5km from the ground at sea level, calculate the approximate travel time for it to hit the ground (use the average of the two velocities and assume no updraft/downdraft with the air).

(Answer: 2.47 m/s, 3.50m/s, 28 minutes)

3. Draw Thiessen polygons on the catchment shown below. If the rainfall depths recorded by Gauge A, B and C are 10mm, 8mm, 7mm and the corresponding polygon areas are 2.1km^2, 9.1km^2 and 2.4km^2, estimate the catchment average rainfall depth and the total volume of water from this rainfall event.

(Answer: 8.5mm, $115.6\times10^3 m^3$)



4. Over a period of 30 years from 1971-2000, records of daily rainfall data have been collected. One site X was inspected in 1985 and a large Willow tree was found to be over-shadowing the gauge. This was cut down in the same year. The data from the gauge was found to be of great potential value in a subsequent reservoir study and a means for inspecting and adjusting the data was sought.

   Use the double mass analysis technique to carry out the following operations using the data in the table:

   a) determine the approximate date of the first significant evidence for over-shadowing of Gauge X;
   b) does the felling of the tree appear to have solved the gauging problem?
   c) evaluate a correction ratios that can be used to adjust incorrect values. (Hint: use graph paper or excel to solve the question)

Year	Gauge X	Other gauge average	Year	Gauge X	Other gauge average
1971	700	510	1986	640	620
1972	550	520	1987	720	360
1973	480	490	1988	510	690
1974	810	620	1989	880	600
1975	430	640	1990	590	580
1976	910	610	1991	710	470
1977	440	550	1992	560	720
1978	890	1110	1993	770	640
1979	470	680	1994	780	660
1980	300	640	1995	770	540
1981	420	620	1996	790	850
1982	430	770	1997	680	630
1983	350	800	1998	340	330
1984	330	710	1999	590	510
1985	880	730	2000	340	340

(Answer: 1978, yes, correction ratio 1.88)

Solutions 2
Precipitation

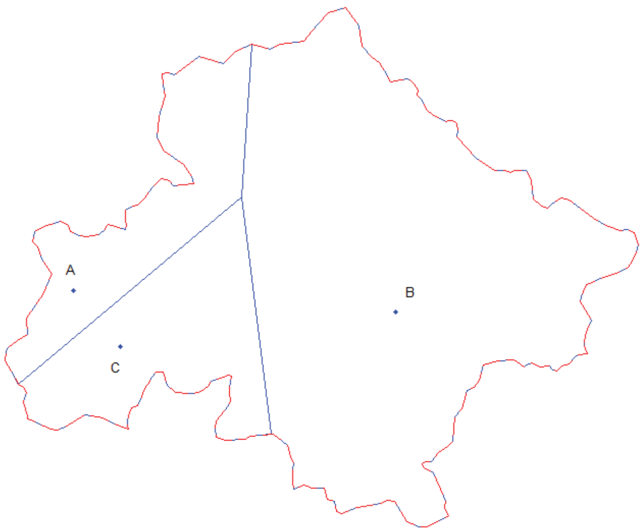
1. Total mass $12.9\times10^{3}\times10^{9}\times1000=12.9\times10^{15}\,kg$

   Total energy $12.9\times10^{15}\times2.5\times10^{6}=32\times10^{21}\,J$

   To replenish the water vapour (about 25 mm of water) from the solar radiation, the time required will be
   $32\times10^{21}/(3.84\times10^{24})$ year=0.008 yr = 2.9 days

2. For a rain drop of 0.6mm in diameter, its terminal velocity is

$$V=\sqrt{\frac{4gD}{3C_d}\left(\frac{\rho_w}{\rho_a}-1\right)}=\sqrt{\frac{4g\times0.6/1000}{3\times1.07}\left(\frac{1000}{1.2}-1\right)}=2.47m/s$$



A higher altitude, the air is thinner, hence less buoyancy and resistance.

$$V=\sqrt{\frac{4gD}{3C_d}\left(\frac{\rho_w}{\rho_a}-1\right)}$$
$$=\sqrt{\frac{4g\times0.6/1000}{3\times1.07}\left(\frac{1000}{1.2/2}-1\right)}=3.50m/s$$

The average velocity is (2.47+3.5)/2=2.99m/s

The travel time = 5000/2.99/60=28 minutes

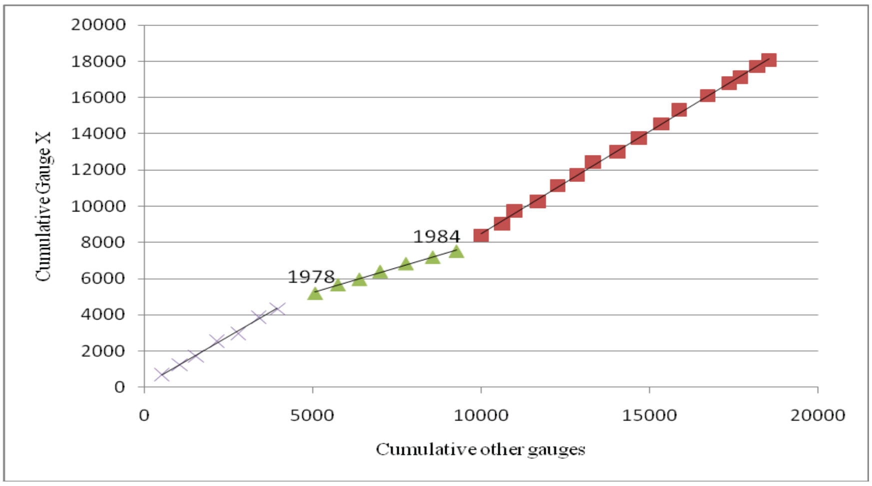
3. Total area = 2.1+9.1+2.4 = 13.6 km^2

$$\bar{R} = \frac{10\times 2.1 + 8\times 9.1 + 9\times 2.4}{13.6} = 8.5mm$$

Total volume =

$$13.6\times 10^6 \times 8.5/1000 = 115.6\times 10^3\, m^3$$

4. From the double mass curve, it can be observed that the period from 1978 - 1984 has different slope, hence the tree influence started from 1978. The felling of the tree appear to have solved the gauging problem. To correct the data from 1978 - 1984,



The ratio of Gauge to other gauges (1978-1984) is $\frac{\text{Gauge X average}}{\text{Other Gauge Average}} = \frac{455.7}{761} = 0.598$,

The ratio in other years $\frac{\text{Gauge X average}}{\text{Other Gauge Average}} = \frac{646.5}{574.0} = 1.126$, the correction ratio is $\frac{1.126}{0.598} = 1.88$

3. Evaporation and Evapotranspiration

Evaporation is the vaporisation of liquid water. Evaporation is an essential part of the water cycle. Solar energy drives evaporation of water from oceans, lakes, moisture in the soil, and other sources of water. In hydrology, evaporation estimation is divided into two categories: evaporation from open water surface and evaporation from land.

3.1 Relevant Basic Terms

3.1.1 Flux

Flux is flow rate divided by the area, i.e., flow rate in a unit area.

$$\text{flux}=\frac{\text{flow rate}}{\text{area}} \tag{1}$$

3.1.2 Radiation emission

Radiation is continuously emitted from all bodies at rates linked with their surface temperature.

$$R_e = \varepsilon\sigma(T+273.15)^4 \tag{2}$$

where R_e is the emitted energy flux (W/m^2), ε is the emissivity of the surface, σ is the Stefan-Boltzmann constant (5.67×10^{-8} $\mathrm{W/m^2\cdot K^4}$) and T is the surface temperature in degrees Celsius. For a perfect radiator (i.e., black body), the emissivity is ε =1. Water's ε = 0.98, sand 0.9 and soil 0.9 ~0.98.

Practice 1

Estimate the radiation from a human body (skin area of 2m^2, skin temperature of 33^{o}C and emissivity =1)

Solution

$$P_{emit} = AR_e = 2\times5.67\times10^{-8}(273.15+33)^4 = 996W$$

Net radiation (with 20^{o}C environment)

$$P_{emit} - P_i = A[R_e - R_i]$$
$$= 2\times5.67\times10^{-8}\left[(273.15+33)^4-(273.15+20)^4\right]$$
$$= 158W$$

3.1.3 Net radiation

When radiation strikes a surface, it is partially reflected and partially absorbed. The reflected fraction is called albedo α $(0 \le \alpha \le 1)$. Deep water absorbs most of the incident radiation with $\alpha \approx 0.06$. Fresh snow's albedo can reach 0.9. The net radiation flux R_n is the difference between the radiation absorbed and emitted.

$$R_n = R_i(1-\alpha) - R_e \tag{3}$$

Eq(3) is applicable to both shortwave and longwave radiations.

3.1.4 Vapour pressure and relative humidity

Water vapour pressure e is the partial pressure contributed by water vapour. When the pressure is in equilibrium, it is called saturated vapour pressure es. The relative humidity is

$$R_h = e / e_s \tag{4}$$

Saturation vapour pressure is related to air temperature,

$$e_s = 611\exp\left(\frac{17.27T}{T+237.3}\right) \tag{5}$$

where e_s is in Pa=N/m^2, T in degrees Celsius.

Δ is the gradient of the saturated vapour pressure curve at air temperature T.

$$\Delta = \frac{de_s}{dT} = \frac{4098e_s}{(T+237.3)^2} \tag{6}$$

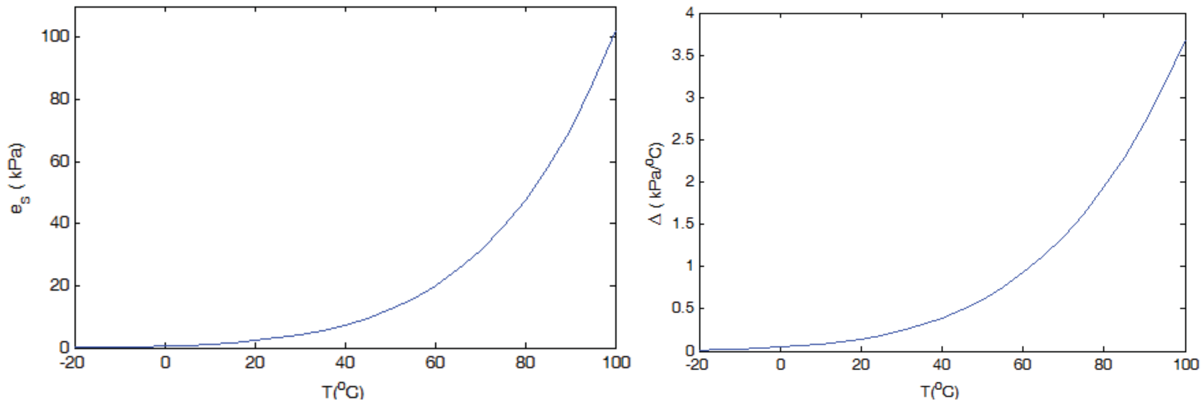


Figure 1 Saturated vapour pressure e_s and its gradient Δ with temperature

3.1.5 Sensible heat

Sensible heat: responsible for liquid water temperature change.

$$\Delta e_u = C_p \Delta T \tag{7}$$

where Δe_u is sensible heat (J/kg), ΔT is temperature change, C_p is the specific heat (water, 4186 J/kg ^{o}C, air 1005 J/kg ^{o}C)

3.1.6 Latent heat

Used to vapourise liquid water into water vapour. It varies slightly with temperature (vaporisation under higher temperature needs less energy)

$$l_v = 2.5\times10^6 - 2370T \quad \text{(J/kg)} \tag{8}$$

where T is temperature in ^{0}C.

3.2 Evaporation from Open Water Surface

Evaporation from open water surface is influenced by two factors: energy input and vapour transport. Energy (mainly solar energy) provides the latent hear for the vapourisation and vapour transport helps to move the vapour away from the water surface.

3.2.1 Energy balance method

The energy input (e.g., solar energy) is used to vapourise liquid water, warm up the water and warm up the underlying soil. If the vapour transport is sufficient (i.e., not a limiting factor), the evaporation rate is

$$E_r = \frac{1}{l_v\rho_w}\left(R_n - H_s - G\right) \tag{9}$$

where E_r is evaporation rate (m/s), H_s is sensible heat flux (in W/m^2, to change liquid water temperature), G is the ground heat flux (in W/m^2, to change underlying soil temperature), R_n is the net radiation flux (W/m^2), l_v is the latent heat of vapourisation (J/kg), ρ_w is water specific density (kg/m^3).

Practice 2
Estimate the evaporation rate (in mm/day) from an open water surface based on the energy balance method. The net radiation is 1000 W/m^2 and air temperature is 20^{0}C. Assume no sensible heat or ground heat flux. The water density is 1000kg/m^3.

Solution
The latent heat at 20^{0}C is $l_v = 2.5\times10^6 - 2370\times20 = 2.45\times10^6\ J/kg$

Hence, the evaporation rate is

$$E_r = \frac{1}{2.45\times10^6\times10^3}(1000-0-0) = 4.08\times10^{-7}\,m/s$$
$$= 35mm/day$$

3.2.2 Aerodynamic method

In addition to the energy, vapour transport is also important. The transport rate is governed by the humidity gradient in the air near the surface and the wind speed across the surface. It is not straightforward to derive a general formula and many forms have been proposed depending on the different assumptions. A commonly used formula is

$$E_a = Du_2(e_{os} - e_{2a})/p \tag{10}$$

where D is a coefficient linked with air and water vapour densities and von Karman constant (see Chow, et al. 1998), u_2 is wind speed at 2m height, p is air pressure, e_{os} is saturated vapour pressure at water surface temperature, e_{2a} is the actual vapour pressure of air (at 2m height). $(e_{os} - e_{2a})$ is termed vapour pressure deficit. It can be seen that evaporation increases when wind speed and vapour pressure deficit increase. It decreases when air pressure goes up.

In practice, a formula derived from Lake Hefner may be used to estimate evaporation from a lake

$$E_{a\_Hefner} = \frac{0.00291}{A^{0.05}} u_2 (e_{os} - e_{2a}) \tag{11}$$

where E_{a_Hefner} is evaporation rate in mm/day based on Hefner study, A is the water surface area (m^2), e_{os} and e_{2a} are as previously defined (Pa), u_2 is in m/s. More empirical equations can be found in Viessman and Lewis (1996).

3.2.3 Combined method

The energy balance method may be used when transport is not limiting and the aerodynamic method is used when energy supply is not limiting. In reality, both factors may be limiting and a combined method should be used.

$$E = \frac{\Delta}{\Delta + \gamma} E_r + \frac{\gamma}{\Delta + \gamma} E_a = \frac{\Delta E_r + \gamma E_a}{\Delta + \gamma} \tag{12}$$

where γ is the psychrometric constant (it represents a balance between the sensible heat gained from air flowing past a wet bulb thermometer and the sensible heat converted to latent heat) and Δ is the gradient of the saturated vapour pressure curve at air temperature (see Eq(6)). The psychrometric constant can be derived as

$$\gamma = \frac{C_p p}{0.622 l_v} \tag{13}$$

where γ is in Pa °C-^{1}, C_p is specific heat of air (1005 J/kg ^{o}C) and l_v latent heat of water, p is atmospheric pressure which is derived from the elevation above sea level,

$$p = 101.3 \left(\frac{293 - 0.0065z}{293} \right)^{5.26} \tag{14}$$

p is in kPa, z is elevation above sea level in m.

3.3 Evapotranspiration from Land

On land, evapotranspiration is a combination of evaporation from the soil surface and transpiration from vegetation. In addition to energy and water transport, the availability of soil water is also important. When water availability is not a limiting factor, evapotranspiration reaches its full potential and is called potential evapotranspiration. In practice, a value for the potential evapotranspiration is calculated at a local climate station on a reference surface (short grass, see FAO 1998). This value is called the reference evapotranspiration, and can be converted to a potential evapotranspiration by multiplying with a surface coefficient. In agriculture, this is called a crop coefficient. As the soil dries out, the rate of evapotranspiration drops below the potential evapotranspiration rate.

The aforementioned combination method was further developed by many researchers and extended to vegetated surfaces by introducing resistance factors. The daily reference evapotranspiration recommended FAO (based on the Penman-Monteith Equation) is

$$ET_0 = \frac{0.408\Delta R_n + \gamma \frac{0.9}{T+273} u_2 \left(e_s - e_a\right)}{\Delta + \gamma\left(1 + 0.34 u_2\right)} \tag{15}$$

where ETo is reference evapotranspiration (mm/day), R_n is net radiation at the grass surface (MJ /m^2 day), T is air temperature at 2 m height (°C), u_2 is wind speed at 2 m height (m/s), e_s is saturation vapour pressure at temperature T (Pa), e_a is actual vapour pressure at temperature T (Pa), Δ is slope vapour pressure curve (Pa/ °C), γ is psychrometric constant (Pa/ °C).

The FAO Penman-Monteith equation determines the evapotranspiration from the hypothetical grass reference surface and provides a standard to which evapotranspiration in different periods of the year or in other regions can be compared and to which the evapotranspiration from other vegetations can be related.

Actual evapotranspiration depends on the vegetation type and availability of soil water. If soil water is not a limiting factor, actual evapotranspiration for a vegetation cover (called crop in the FAO report) is

$$ET_c = K_c ET_0 \tag{16}$$

where ET_c crop evapotranspiration (mm/day), K_c crop coefficient (dimensionless), ET_0 reference crop evapotranspiration (mm/day). A list of KC values can be found in FAO 1998 (p127).

Most of the effects of the various weather conditions are incorporated into the ET_0 estimate. Therefore, as ET_0 represents an index of climatic demand, K_c varies predominately with the specific crop characteristics and only to a limited extent with climate. This enables the transfer of standard values for K_c between locations and between climates.

3.4 Field measurements

3.4.1 Pan

An evaporation pan is used to hold water during observations for the determination of the quantity of evaporation at a given location. Often the evaporation pans are automated with water level sensors and a small weather station is located nearby. Pan evaporation is used to estimate the evaporation from lakes and land. Evaporation from a natural body of water is usually at a lower rate because the body of water does not have metal sides that get hot with the sun.

$$ET_0 = K_p E_{pan} \tag{17}$$

where ET_o reference evapotranspiration (mm/day), K_p pan coefficient, usually taken as 0.75. $_{Epan}$ pan evaporation (mm/day).

3.4.2 Lysimeter

A lysimeter is a measuring device which can be used to measure the amount of actual evapotranspiration which is released by plants, usually crops or trees. By recording the amount of precipitation that an area receives and the amount lost through the soil, the amount of water lost to evapotranspiration can be calculated. It does this by isolating the vegetation root zone from its environment and controlling the processes that are difficult to measure, the different terms in the soil water balance equation can be determined with greater accuracy. A requirement of lysimeters is that the vegetation both inside and immediately outside of the lysimeter be perfectly matched (same height and leaf area index). This requirement has historically not been closely adhered to in a majority of lysimeter studies and has resulted in severely erroneous and unrepresentative data. As lysimeters are difficult and expensive to construct and as their operation and maintenance require special care, their use is limited to specific research purposes (FAO, 1998).

3.4.3 Eddy covariance

This is a prime atmospheric flux measurement technique to measure and calculate vertical turbulent fluxes within atmospheric boundary layers. It is a statistical method that analyses high-frequency wind and scalar atmospheric data series, and yields values of evaporation or evapotranspiration. The technique is mathematically complex, and requires significant care in setting up and processing data (Wikipedia, 2009).

3.4.4 Catchment/reservoir water balance

Evapotranspiration may be estimated by creating an equation of the water balance of a catchment. The equation balances the change in water stored within the basin (S) with precipitation P, surface runoff R, groundwater runoff G and storage change ΔS .

$$ET = P - R - G - \Delta S \tag{18}$$

For annual time step, the storage change may be ignored, so $\Delta S = 0$

Questions 3

Evaporation and Evapotranspiration

1. What weather variables are needed for calculating evaporation from open water surface with the combined method?

2. What are the factors that influence the actual evapotranspiration on land?

3. What are the potential evapotranspiration and reference evapotranspiration? Describe their relationship with the actual evapotranspiration.

4. Using the Hefner equation, find the daily evaporation rate (in mm/day) for a lake of area 5 km^2 given that the mean air temperature is 20^oC and water surface temperature is 15^0C. The average wind speed is 15 km/h, and relative humidity is 20% (all the measures in air are at 2m height). If the same evaporation rate is maintained for a whole year, how much water is lost due to evaporation (m^3)?

   (Answer: 7.00 mm/day, 12.8 million m^3)

5. With the same lake in Q4, if the net radiation is 210W/m^2 and the lake is 1000m above sea level, estimate evaporation rate (in mm/day) using the combined method (assume water density is 1000kg/m^3).

   (Answer: 7.04 mm/day)

6. With the same weather conditions as Q4 and Q5 (replacing the lake with a land), estimate reference evapotranspiration using FAO Penman- Monteith equation.

   (Answer: 5.90 mm/day)

Solutions 3

Evaporation and Evapotranspiration

4. At water surface,

$$e_{os} = 611\exp\left(\frac{17.27T}{T+237.3}\right) = 611\exp\left(\frac{17.27\times 15}{15+237.3}\right) = 1706Pa$$

In air, saturated water vapour pressure is

$$e_{2s} = 611\exp\left(\frac{17.27T}{T+237.3}\right) = 611\exp\left(\frac{17.27\times 20}{20+237.3}\right) = 2339Pa$$

With 20% relative humidity, the actual water vapour pressure is

$$e_{2a} = R_h e_s = 0.2\times 2339 = 468Pa$$

Wind speed $= 15\times 1000/3600 = 4.2m/s$

Hence

$$E_{a\_Hefner} = \frac{0.00291}{A^{0.05}} u_2 (e_{os} - e_{2a}) = \frac{0.00291}{(5 \times 10^6)^{0.05}} 4.2(1706 - 468)$$
$$= 7.00 mm / day$$

If the same evaporation rate is maintained for a whole year, the water loss will
$0.007 \times 365 \times 5 \times 10^6 = 12.8 \times 10^6 m^3$

5. The latent heat at 20^{0}C is

$$l_v = 2.5 \times 10^6 - 2370T = 2.45 \times 10^6 J / kg$$

Hence, the evaporation rate is

$$E_r = \frac{1}{l_v \rho_w} \left(R_n - H_s - G \right) = \frac{1}{2.45 \times 10^6 \times 1000} \left(200 - 0 - 0 \right)$$
$$= 81.6 \times 10^{-9} m / s = 7.05 mm / day$$

The saturated water vapour gradient at 20^{o}C is

$$\Delta = \frac{4098 e_s}{\left(T + 237.3 \right)^2} = \frac{4098 \times 2339}{\left(20 + 237.3 \right)^2} = 145 Pa /^o C$$

The air pressure at 1000 above sea level is

$$p = 101.3 \left(\frac{293 - 0.0065z}{293} \right)^{5.26} = 101.3 \left(\frac{293 - 0.0065 \times 1000}{293} \right)^{5.26}$$
$$= 90.0 kPa = 90 \times 10^3 Pa$$
so

$$\gamma = \frac{C_p p}{0.622 l_v} = \frac{1005 \times 90 \times 1000}{0.622 \times 2.45 \times 10^6} = 59 Pa /^o C$$

To combine them

$$E = \frac{\Delta E_r + \gamma E_a}{\Delta + \gamma} = \frac{145 \times 7.05 + 59 \times 7}{145 + 59} = 7.04 mm / day$$

There is not much difference between the three methods in this case.

6. From the same weather conditions, so

$\Delta = 145 Pa / ^o C$, $u_2 = 4.2 m / s$, $\gamma = 59 Pa / ^o C$,
$T = 20\ ^oC$, $e_a = 468 Pa$, $e_s = 2339 Pa$

$$R_n = 210 W / m^2 = 210 \times 3600 \times 24 / 10^6 MJ / m^2 \cdot day$$
$$= 18.1 MJ / m^2 \cdot day$$

so

$$ET_0 = \frac{0.408 \Delta R_n + \gamma \frac{0.9}{T + 273} u_2 \left(e_s - e_a \right)}{\Delta + \gamma \left(1 + 0.34 u_2 \right)}$$
$$= \frac{0.408 \times 145 \times 18.1 + 59 \frac{0.90}{20 + 273} 4.2 \left(2339 - 468 \right)}{145 + 59 \left(1 + 0.34 \times 4.2 \right)}$$
$$= \frac{1071 + 339}{288} = 5.9 mm / day$$

4. Infiltration

Infiltration is the process of water penetrating from the ground surface into the soil. The maximum rate at which water can enter the soil is called the infiltration capacity.

4.1 Relevant Basic Terms

4.1.1 Porosity

$$\eta = \frac{\text{volume of voids}}{\text{total volume}} \tag{1}$$

The range for soil η is usually around 0.25 ~ 0.75.

4.1.2 Soil moisture content

The voids in soil are occupied by liquid water and air. The soil moisture content describes the liquid water volume in the soil.

$$\theta = \frac{\text{volume of liquid water}}{\text{total volume}} \tag{2}$$

Hence $0 \le \theta \le \eta$. For dry soil, $\theta = 0$ and saturated soil $\theta = \eta$

4.1.3 Vadose zone (unsaturated zone)

It is between the land surface and the water table. Water in the vadose zone has a pressure head less than atmospheric pressure (called the suction head).

4.1.4 Field capacity

It is the amount of soil liquid moisture held in soil after excess water has drained away and serves as a measure of soil water-holding capacity. In practice, the field capacity is found after the saturated soil is drained for 2 - 3 days. The soils in England during the winter are around field capacity since precipitation exceeds evapotranspiration.

4.1.5 Soil moisture deficit (SMD)

This represents the amount of rainfall necessary to return the soil to 'field capacity'.

4.1.6 Darcy’s law (saturated soil)

It is used to describe the flux q to the head loss per unit length of medium S_f

$$q = KS_f = K\frac{\Delta h}{\Delta s} \tag{3}$$

where K is hydraulic conductivity (mm/s). The total head $h = \dfrac{p}{\rho g} + z$

4.1.7 Pore velocity in soil

The pore velocity is related to the Darcy flux (q) by the porosity. The flux is divided by porosity to account for the fact that only a fraction of the total soil volume is available for flow.

$$V = \frac{q}{\eta} \tag{4}$$

4.1.8 Darcy’s law (unsaturated soil)

Total head is the suction head and gravity head.

$$h = \Psi + z \text{ and } q = K(\theta) S_f = -K(\theta)\frac{\Delta h}{\Delta z} \qquad (5)$$

The Darcy's law is still applicable. The difference is that the hydraulic conductivity is not constant anymore and varies with soil moisture content (Figure 1). The drier the soil, the smaller the hydraulic conductivity. On the other hand, the soil suction head's absolute value goes up with drier soil. z should be negative and measured from the ground surface.

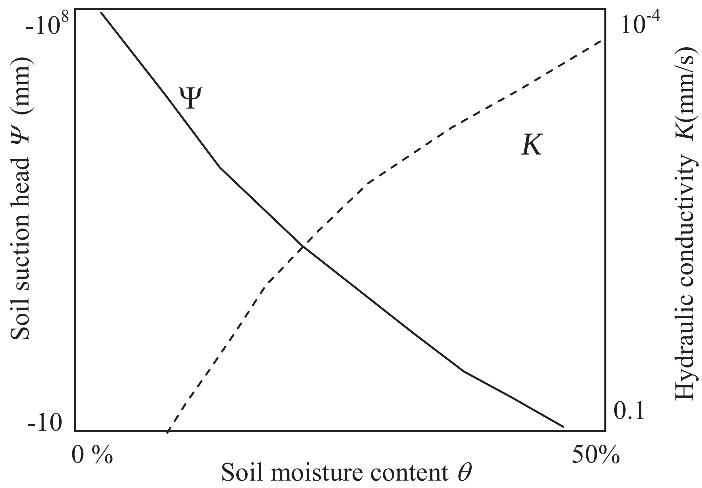


Figure 1 Illustration of soil suction head and hydraulic conductivity with soil moisture content

4.2 Infiltration Process

When water is ponded on a homogeneous soil, characteristic zones of saturation, water transmission, and soil wetting, develop as the wetting front propagates downward.

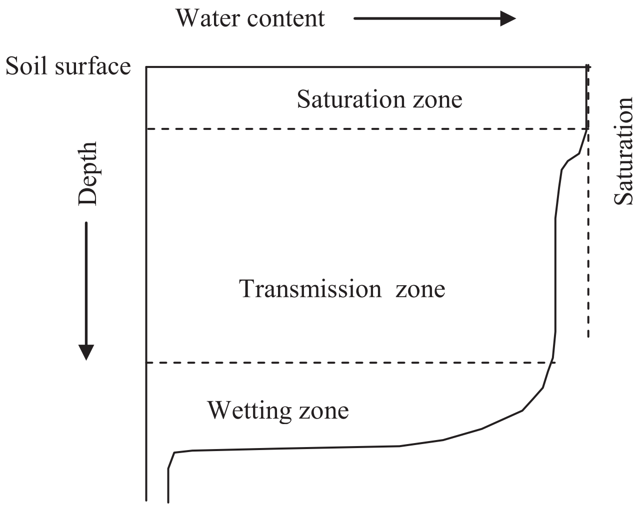


Figure 2 Moisture zones during infiltration

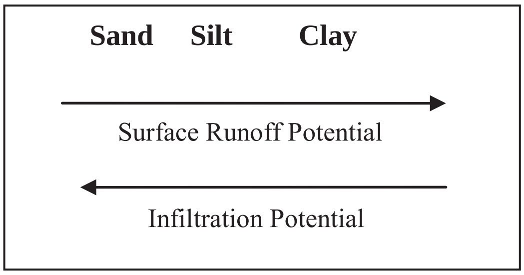


Figure 3 Effect of soil types

The factors affecting infiltration are:

1) precipitation;
2) soil types;
3) water contents in the soil;
4) vegetation cover;
5) ground slope

4.3 Estimation of Infiltration Rate

4.3.1 Horton's Equation (1940)

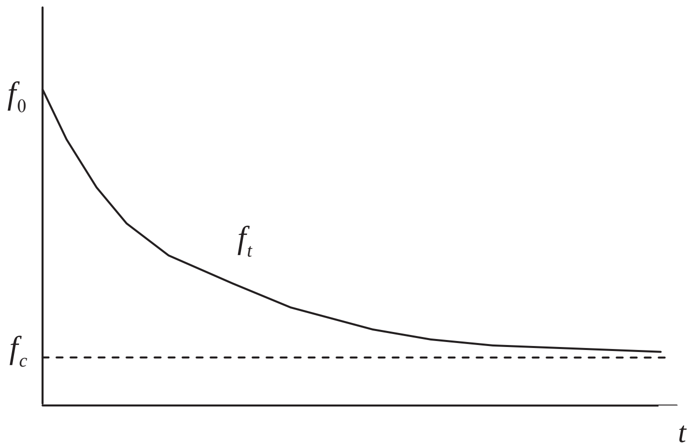


Figure 4 Horton's infiltration curve

$$f_t = f_c + (f_0 - f_c)e^{-kt} \tag{6}$$

where f_t is infiltration capacity at time t (mm/hr), f_0 is initial infiltration capacity (mm/hr), f_c is final capacity (mm/hr), k is empirical constant (hr^{-1}).

Total infiltration in *T* hour period

$$F = \int_0^T f_t dt = \int_0^T f_c + (f_0 - f_c)e^{-kt} dt$$
$$= \left[f_c t - (f_0 - f_c)e^{-kt}/k \right]_0^T = f_c T - (f_0 - f_c)e^{-kT}/k + (f_0 - f_c)/k$$
$$= f_c T + \frac{1}{k}(f_0 - f_c)\left(1 - e^{-kT}\right) \tag{7}$$

Practice 1

The initial infiltration capacity f_0 of a catchment is estimated as 4.5mm/hr, the time constant as 0.35/hour, and the capacity f_c as 0.4 mm/hour. Use Horton's equation to find a) the value f_t at t=10 min, 30 min, 1 hr, 2hr and 6 hr; b) the total volume of infiltration over the 6 -hr period. Assume continuously ponded conditions.

Solution

From Horton's equation

$$f_t = f_c + (f_0 - f_c)e^{-kt} = 0.4 + (4.5 - 0.4)e^{-0.35t}$$

Hence, the answers are

t (hr)	1/6	½	1	2	6
ft (mm/hr)	4.3	3.8	3.3	2.4	0.90

Integrating over the interval [0, 6] gives:

$$F = f_c T + \frac{1}{k}(f_0 - f_c)\left(1 - e^{-kT}\right)$$
$$= 0.4 \times 6 + \frac{1}{0.35}\left(4.5 - 0.4\right)\left(1 - e^{-0.35 \times 6}\right)$$
$$= 12.7mm$$

For Horton's equation, the infiltration curve may be different due to different initial soil moisture contents.

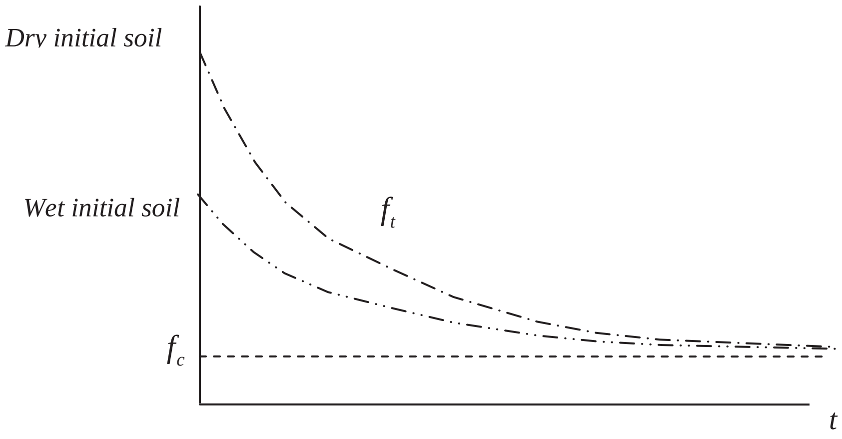
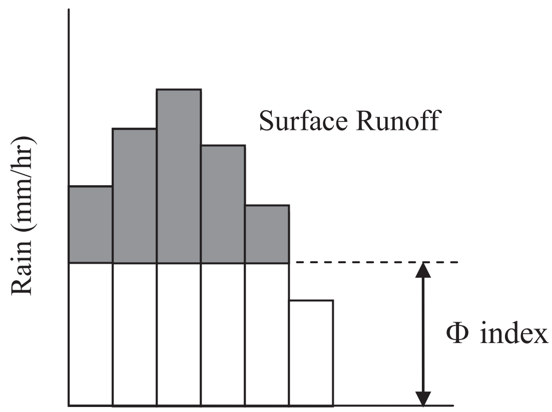


Figure 5 Different Horton's infiltration curves due to initial soil moisture contents

4.3.2 Φ index

It assumes no variation with f_t with time (i.e., constant infiltration capacity).



Practice 2
A catchment of are 0.25 km^2 is subjected to a storm with the following profile:

Time (hr)	1	2	3	4	5	6
Rain (mm)	7	18	25	12	10	3

If the volume of storm runoff is 8250m^3, estimate the index (neglect the effect of ET)

Solution
Total runoff in mm

$$\frac{8250m^3}{Area} = \frac{8250m^3}{0.25\times10^6 m^2} = 0.033m = 33mm$$

Water balance

$$\{7-\Phi\}+\{18-\Phi\}+\{25-\Phi\}+\{12-\Phi\}+\{10-\Phi\}+\{3-\Phi\}=33mm$$

Solve it with

$$\Phi = 7mm$$

Since each $\{\ \}$ should be ≥ 0, neglect $\{7-\Phi\}$ and $\{3-\Phi\}$ which didn't contribute to the runoff
A new balance will be

$$\{18-\Phi\}+\{25-\Phi\}+\{12-\Phi\}+\{10-\Phi\}=33mm$$

Solve it with

$$\Phi = 8mm$$

Check $\{\ \}\geq 0$, so the final $\Phi =$ 8 mm is the right answer.

4.3.3 Green-Ampt method

The Green-Ampt infiltration model is widely used in some engineering models (e.g., SWMM, HEC-HMS). It can consider the initial soil moisture condition.

$$f_t = K\left[\frac{1+(\eta-\theta_i)\Psi_f}{F_t}\right] \tag{8}$$

where K is saturated hydraulic conductivity, θ_i is initial soil moisture content, $(\eta-\theta_i)$ is volume moisture deficit, Ψ_f is wetting front suction. F_t is cumulative infiltration at time t.

4.4 Infiltration measurements

4.4.1 Infiltrometer

Infiltrometer is a device used to measure the rate of water infiltration into soil or other porous media. Commonly used infiltrometers are single ring or double ring infiltrometer.

It is easy to use, but soil structure could be disturbed.

4.4.2 Artificial rain simulation

No disturbance to soil, close to true rainfall impact, but high cost.

Questions 4
Infiltration

1. What is the infiltration capacity?

2. With the measurements in the following table, calculate the soil moisture flux q (cm/day) between depth 0.5m and 0.8m in each week. Hydraulic conductivity $K = 240(-\Psi)^{-2.3}$ (K in cm/day and Ψ in cm). Use the average suction head to derive hydraulic conductivity. For $1km^2$ area, how much water has passed the layer between 0.5m and 0.8m in these two weeks (in m^3)?

Week	Total head at 0.5m (cm)	Total head at 0.8m (cm)
1	-70	-105
2	-80	-120

(Answers: -0.2174cm/day, -0.0899cm/day, 21510m^3)

3. Suppose that the parameters for Horton's equation are fo=3.5mm/hr, fc=0.6 mm/hr and k=4.1 hr-1. Determine the infiltration rates after 0, 10 min, 20 min, 1 hr, 1.5 hr and 2 hr and the cumulative infiltration after 2 hours. Assume continuously ponded conditions.

(Answers: 3.50, 2.04, 1.35, 0.648, 0.606, 0.601 mm/hr, the cumulative infiltration 1.91 mm)

4. A catchment of area 200km^2 is subjected to a storm with the following profile

Time (hr)	3	6	9	12	15	18	21	24
Rain (mm)	16.5	48.0	20.0	12.8	9.1	5.5	3.1	1.2

If the volume of storm surface runoff is $1.6\times10^7 m^3$, estimate the Φ index (in mm/hr) (neglect the effect of ET).

(Answer: 1.77 mm/hr)

Solutions 4
Infiltration

2. Total head = suction head + gravity head , so $\Psi = h - z$

The average flux between $z_1 = -50cm$ and $z_2 = -80cm$ is

$$q = KS_f = -K_{mean}\frac{\Delta h}{\Delta z} = K_{mean}\frac{h_1 - h_2}{30}$$

The results are

Week	Total head at 0.5m (cm)	Total head at 0.8m (cm)	Suction head at 0.5m (cm)	Suction head at 0.8m (cm)	K (cm/day)	head difference (cm)	flux (cm/day)
1	-70	-105	-20	-25	0.1863	35	-0.2174
2	-80	-120	-30	-40	0.0674	40	-0.0899

The total flux in two weeks is

$$-0.2174 \times 7 - 0.0899 \times 7 = 2.151cm$$

within 1km^2 $2.151cm / 100m \times 10^6 = 21510m^3$

3. From Horton's equation

$$f_t = f_c + (f_0 - f_c)e^{-kt} = 0.6 + (3.5 - 0.6)e^{-4.1t}$$

Hence, the answers are

t (hr)	0	1/6	1/3	1	1.5	6
ft (mm/hr)	3.50	2.06	1.34	0.648	0.606	0.600

Integrating over the interval [0, 2] gives:

$$F = \int_0^T f_c + (f_0 - f_c)e^{-kt}d = f_cT + \frac{1}{k}(f_0 - f_c)\left(1 - e^{-kT}\right)$$

$$= 0.6 \times 2 + \frac{1}{4.1}\left(3.5 - 0.6\right)\left(1 - e^{-4.1\times 2}\right) = 1.91mm$$

4. Total runoff in mm

$$\frac{1.6 \times 10^7 m^3}{Area} = \frac{1.6 \times 10^7 m^3}{200 \times 10^6 m^2} = 0.08m = 80mm$$

Water balance

$$\{16.5-\Phi\}+\{48-\Phi\}+\{20-\Phi\}+\{12.8-\Phi\}+\{9.1-\Phi\}+\{5.5-\Phi\}+\{3.1-\Phi\}+\{1.2-\Phi\}=80mm$$

Solve it with

$\Phi = 4.51mm/3hr$

Since each $\{\ \}$ should be ≥ 0, neglect $\{3.1-\Phi\}$ and $\{1.2-\Phi\}$ which didn't contribute to the runoff

A new balance will be

$$\{16.5-\Phi\}+\{48-\Phi\}+\{20-\Phi\}+\{12.8-\Phi\}+\{9.1-\Phi\}+\{5.5-\Phi\}=80mm$$

Solve it with

$\Phi = 5.32mm/3hr$

Check $\{\ \}\geq 0$, so the final $\Phi = 5.32mm/3hr = 1.77mm/hr$ is the right answer.

5. Groundwater

Groundwater is the water beneath the ground surface contained in void spaces (pore spaces between rock and soil particles, or bedrock fractures).

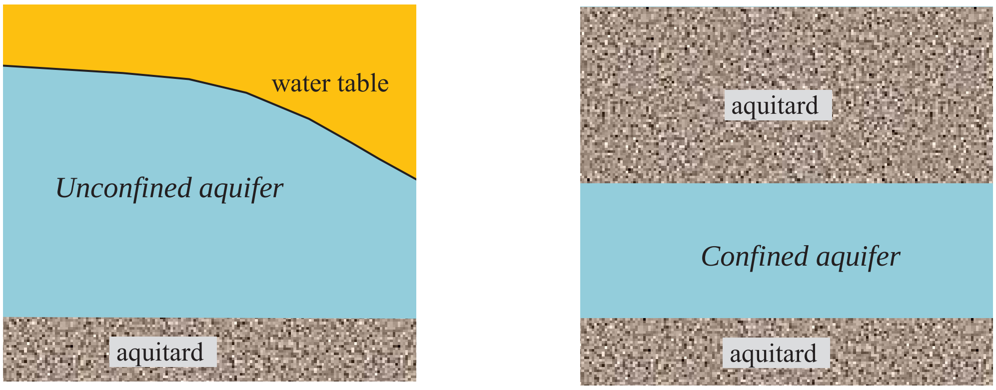


Figure 1 Aquifer notations

5.1 Basic Terms

5.1.1 Aquifer

An aquifer is an underground layer of water-bearing permeable rock or unconsolidated materials (gravel, sand, silt, or clay) from which groundwater can be usefully extracted using a water well.

5.1.2Water table

The water table is the level at which the groundwater pressure is equal to atmospheric pressure.

5.1.3 Aquitard

An aquitard is a zone within the earth that restricts the flow of groundwater from one aquifer to another. Aquitards comprise layers of either clay or non-porous rock with low hydraulic conductivity.

5.1.4 Unconfined aquifer

It is an aquifer with the water table as its upper boundary. Because the aquifer is not under pressure the water level in a well is the same as the water table outside the well.

5.1.5 Confined aquifer

It is an aquifer found between two relatively impermeable layers.

5.1.6 Artesian aquifer/well

It is a confined aquifer containing groundwater that will flow upward through a well, called an artesian well, without the need for pumping. Water may even reach the ground surface if the natural pressure is high enough, in which case the well is called a flowing artesian well.

5.1.7 Water well

A water well is an excavation or structure created in the ground by digging, driving, boring or drilling to access groundwater in underground aquifers.

5.1.8 Borehole

It is a narrow shaft drilled in the ground as part of a groundwater site assessment.

5.1.9 Piezometric surface

The imaginary surface that everywhere coincides with the piezometric head of the water in the aquifer. In areas of artesian ground water, it is above the land surface.

5.1.10 Base flow

Base flow is the portion of stream flow that comes from groundwater. It sustains flows in a river during the dry periods between rainstorms.

5.1.11 Groundwater Recharge

The natural or intentional infiltration (percolation) of surface water into the groundwater system.

5.1.12 Fossil water

Fossil water is groundwater that has remained in an aquifer for thousands or even millions of years. When geologic changes seal the aquifer off from further recharging, the water becomes trapped inside.

5.2 Characteristics of Confined/Unconfined Groundwater

Unconfined groundwater	Confined groundwater
1) Free water table	1) No water table - piezometric surface
2) Hydraulics gradient can change rapidly	2) Hydraulic gradient more uniform
3) Greater fluctuations with seasons	3) Smaller fluctuations with season
4) Drill to water table	4) Drill to aquifer
5) No flowing wells	5) May get flowing wells
6) Recharge area is around borehole	6) Recharge area is away from borehole
7) On pumping, aquifer dewatered	7) On pumping, aquifer not dewatered - pressure decrease over large area
8) More complex mathematics	8) Simpler mathematics
9) Must be below ground surface	9) May be above ground surface

5.3 The Basic Flow Equations

Reynolds number

$$\mathrm{Re} = \frac{\rho q d}{\mu} \tag{1}$$

where q is flux (not pore velocity), d is the mean soil grain diameter, μ is viscosity, ρ is water density

For Darcy's law to be valid, *Re* should be less than 1 (i.e., slow flow and in laminar flow state). In most practical engineering problems, *Re* is less than one.

Practice
Water in an aquifer moves with a flux of 30cm/day. The average soil particle size is 1.5 mm. Find the Reynolds number and check whether Darcy's law is applicable (water density is 1000kg/m^3 and water viscosity is 1.137x10^{-3} Ns/m^2).

Solution
Convert all the units to metre, Newton and second

$$\mathrm{Re} = \frac{1000 \times \left(0.3/24/3600\right) \times 0.0015}{1.137 \times 10^{-3}} = 0.0046$$

Since Re < 1, Darcy's law is applicable.

According to Darcy's law $q = KS_f = K\frac{dh}{ds}$, the flow rate across an area is

$$Q = Aq = AK\frac{dh}{ds} \tag{2}$$

Some K values in nature (m/day) (from 'Connected Water Resources Project', 2009)

Clay 10-2 ~10-8
Sandstone 1~10-3
Sand 2~20
Gravel 100~1000

5.4 Steady Flow

A steady flow means that the flow characteristics (depth, velocity, flow rate, etc.) are not changing with time.

5.4.1 Unconfined flow to a well

Consider an infinite unconfined aquifer of constant thickness and permeability. Flow towards the well is radial and entirely horizontal. The well is a fully penetrating type.

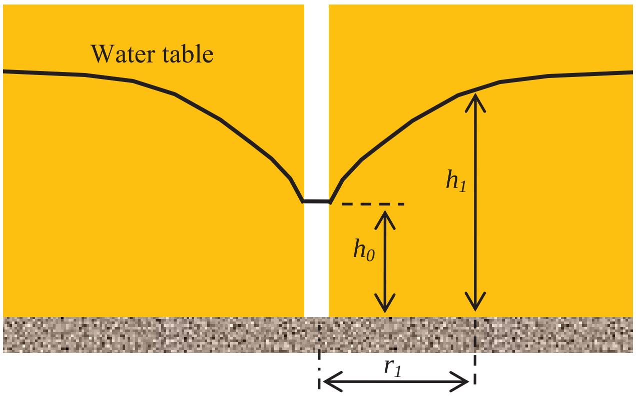


Figure 2 Unconfined flow to a well

From Darcy's law

$$Q = AK\frac{dh}{ds}$$

In Polar coordinate

$$Q = 2\pi rhK\frac{dh}{dr}$$

Rearrange

$$hdh = \frac{Q}{2\pi rK}dr$$

Integrate between the well wall (r_0) to r_1

$$\int_{h_0}^{h_1} h\,dh = \int_{r_0}^{r_1} \frac{Q}{2\pi r K}\,dr$$

So

$$\frac{1}{2}\left[h^2\right]_{h_0}^{h_1} = \frac{Q}{2\pi K}\left[\ln r\right]_{r_0}^{r_1} \quad \text{i.e.} \quad \frac{1}{2}\left(h_1^2 - h_0^2\right) = \frac{Q\ln(r_1/r_0)}{2\pi K}$$

Hence

$$Q = \frac{\pi K\left(h_1^2 - h_0^2\right)}{\ln(r_1/r_0)} \tag{3}$$

5.4.2 Confined flow to a well

Consider an infinite confined aquifer of constant thickness and permeability. Flow towards the well is radial and entirely horizontal. The well is a fully penetrating type.

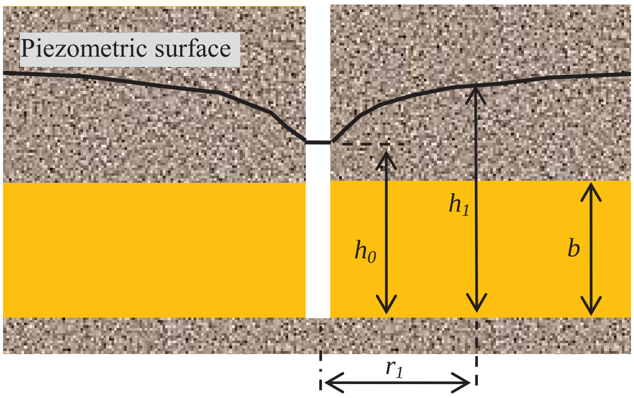


Figure 3 Confined flow to a well

From Darcy's law

$$Q = AK\frac{dh}{ds}$$

In Polar coordinate

$$Q = 2\pi rbK\frac{dh}{dr}$$

Rearrange

$$dh = \frac{Q}{2\pi rbK}dr$$

Integrate between the well wall (r_0) to r1

$$\int_{h_0}^{h_1} dh = \int_{r_0}^{r_1} \frac{Q}{2\pi rbK}dr$$

So

$$[h]_{h_0}^{h_1} = \frac{Q}{2\pi bK}[\ln r]_{r_0}^{r_1} \quad \text{i.e.} \quad \left(h_1 - h_0\right) = \frac{Q\ln(r_1/r_0)}{2\pi bK}$$

Hence

$$Q = \frac{2\pi bK\left(h_1 - h_0\right)}{\ln(r_1/r_0)} \qquad (4)$$

For an aquifer of thickness b, transmissivity is defined as

$$T = bK \quad (\mathrm{m^2/day}) \tag{5}$$

So

$$Q = \frac{2\pi T\left(h_1 - h_0\right)}{\ln(r_1 / r_0)} \tag{6}$$

At a distance R (Radius of Influence) the piezometric surface is almost constant at H. this decides the limit on Q for a given well diameter.

5.5 Unsteady Flow

For unsteady groundwater flow, the two dimensional equation under unconfined aquifer conditions with a flat bed is called the Boussinesq equation

$$\frac{\partial h}{\partial t} = \frac{K}{\mu}\left[\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y}\right)\right] + \frac{P}{\mu} \tag{7}$$

where P is the percolated precipitation (or evapotranspiration loss).

This equation has unknown h and its second order derivatives. The general forms of unsteady groundwater flow equations are more complicated. They are not solvable by analytical methods in most practical cases. The advancement in computing technology has provided numerical solutions to such a problem.

5.6 Computer Software

5.6.1 MODFLOW

MODFLOW is the U.S. Geological Survey modular finite-difference flow model, which is a computer code that solves the groundwater flow equation. The program is used by engineers and scientist to simulate the flow of groundwater through aquifers. The code is free software, written primarily in Fortran, and can compile and run on DOS, Windows or Unix-like operating systems. Since its original development in the early 1980s, the USGS have released four major releases, and is now considered to be the de facto standard code for aquifer simulation.

The latest version is MODFLOW-2005 Version 1.7 released on 23 September 2009.

See http://water.usgs.gov/nrp/gwsoftware/modflow2005/modflow2005.html

6.2 FEFLOW

FEFLOW (Finite Element subsurface FLOW system) is a commercial computer program for simulating groundwater flow, mass transfer and heat transfer in porous media. The program uses finite element analysis to solve the groundwater flow equation of both saturated and unsaturated conditions as well as mass and heat transport, including fluid density effects and chemical kinetics for multi-component reaction systems. It is owned by DHI Group (Danish Hydraulics Istitute). FEFLOW has a unique graphical user interface when compared to most other Microsoft Windows-based groundwater modelling software. This is attributed to the Motif widget toolkit used to develop the interface. The program is offered in both 32-bit and 64-bit versions for Linux and for Microsoft Windows operating systems (using a proprietary X Window System, Hummingbird Exceed). Support for UNIX platforms ended with version 5.3; prior support was provided for IRIX, Tru64, and Solaris. The latest version is FEMFLOW 5.4.

See http://www.feflow.info/

6.3 MIKE SHE

MIKE SHE is a commercial computer program owned by DHI and is an integrated hydrological modelling system for building and simulating surface water flow and groundwater flow. MIKE SHE can simulate the entire land phase of the hydrologic cycle and allows components to be used independently and customized to local needs. MIKE SHE emerged from System Hydrologique European (SHE) as developed and extensively applied since 1977 onwards by a consortium of three European organizations: the Institute of Hydrology (the United Kingdom), SOGREAH (France) and DHI Water.Environment.Health(Denmark). Then DHI has continuously put effort in development and research on MIKE SHE. MIKE SHE can be used for the analysis, planning and management of a wide range of water resources and environmental problems related to surface water and groundwater, especially surface water impact from groundwater withdrawal, conjunctive use of groundwater and surface water, wetland management and restoration, river basin management and planning, impact studies for changes in land use and climate. The program is offered in both 32-bit and 64-bit versions for Microsoft Windows operating systems.

See http://www.dhigroup.com/Software/WaterResources/MIKESHE.aspx

Questions 5

Groundwater

a) What are the essential differences between unconfined and confined groundwater flow?

b) A well of 0.3m radius in a confined aquifer was pumped at a steady rate of 30litre/s from a fully penetrating well. When the well level remained constant at 85.5m above datum, the observation well (borehole) constructed at a distance of 10m recorded a water level of 86.5m. The aquifer thickness is estimated at 20m.

1) What are the hydraulic conductivity and transmissivity of the aquifer around the well (in m/day and m^2/day)?

(Answers: 72.3 m/day, 1452 m^2/day)

2) If a small village is about 300m away from the new production well, estimate the impact of the well to the piezometric surface at the village if the piezometric surface prior to the pumping is about 90m AOD.

(Answer: level drop by 2.5 m)

You are required to work out the solution from Darcy's law, instead of using Eq(4) directly.

Solutions 5

Groundwater

1. From Darcy's law

$$Q = AK\frac{dh}{ds}$$

In Polar coordinate

$$Q = 2\pi rbK\frac{dh}{dr}$$

Rearrange

$$dh = \frac{Q}{2\pi rbK}dr$$

Integrate between the well wall (r_0) to r_1

$$\int_{85.5}^{86.5} dh = \int_{0.3}^{10} \frac{0.03}{2\pi \times 20Kr} dr$$

So

$$[h]_{85.5}^{86.5} = \frac{2.39\times10^{-4}}{K}[\ln r]_{0.3}^{10} \quad \text{i.e.} \quad 1 = \frac{2.39\times10^{-4}\ln(10/0.3)}{K}$$

Hence

$$K = 8.38\times10^{-4} m/s = 72.3 m/day$$

For an aquifer of thickness b, transmissivity is defined as

$$T = bK = 20\times8.4\times10^{-4} = 0.0168 \text{ m}^2\text{/s} = 1452 \text{ (m}^2\text{/day)}$$

2. From the integration,

$$\int_{85.5}^{h_1} dh = \int_{0.3}^{300} \frac{0.03}{2\pi\times20\times8.4\times10^{-4} r} dr$$

$$h_1 - 85.5 = 0.2842[\ln r]_{0.3}^{300}$$

$$h_1 - 85.5 = 0.2842\ln(300/0.3)$$

$$h_1 = 87.5m$$

so the piezometric surface drop at the village is 90-87.5= 2.5m

6. Hydrograph

A hydrograph is a graph showing changes in the discharge of a river over a period of time. It represents how a catchment responds to rainfall (Figure 1).

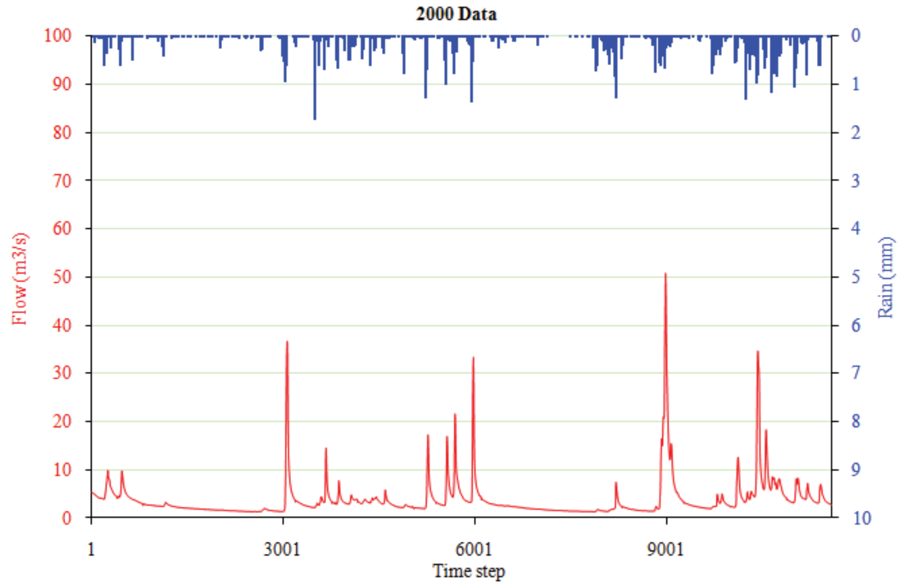


Figure 1 Hydrograph of the River Brue, England

6.1 Basic Terms

6.1.1 River Runoff

River runoff is surface water in rivers. When rain falls onto the earth, it starts to move according to the law of gravity. There are four ways that rainfall contributes to river runoff: 1) overland surface runoff; 2) interflow (subsurface runoff), 3) base flow from groundwater; 4) rainfall onto river channel.

6.1.2 Infiltration excess runoff

When rainfall intensity is greater than ground surface infiltration rate, surface runoff is generated. Infiltration excess flow usually occurs during high intensity rainfall events. If the subsoil infiltration rate is lower than the top soil infiltration rate, infiltration excess runoff can occur below the ground surface and this will result in interflow (i.e., subsurface runoff).

6.1.3 Saturation excess runoff

When rainfall intensity is less than soil infiltration rate, prolonged rainfall will saturate the soil and no more water could be held. As a result, excess water is released from the soil into groundwater and river channels.

6.1.4 Direct runoff

Overland surface runoff and interflow travel much faster than groundwater, hence they are combined into a term 'direct runoff' in hydrology.

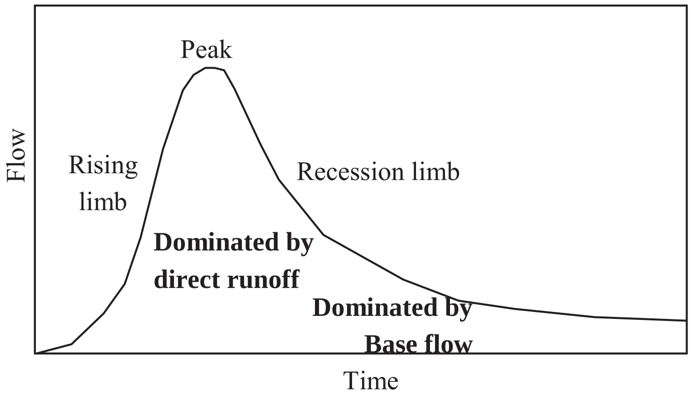


Figure 2 Hydrograph components

6.1.5 Hydrograph components

A hydrograph has a rising limb, a recession limb and a peak flow. The division between direct runoff and base flow is transitional and not very clear (Figure 2).

6.2 Flow Event Separation

In water balance calculations, it is necessary to separate individual rainfall runoff events as shown Figure 3. The shaded area is the total flow volume, which is not easy to calculate since the recession curve of this event will reach infinity with time. In practice, a recession curve formula is used to estimate the water volume.

If the recession part is dominated by groundwater supply, the recession curve can normally be approximated as

$$Q_t = Q_0 e^{-t/k} \tag{1}$$

where k is exponent, Q_0 is discharge at time zero (it can be set anywhere in the lower part of the recession limb), Q_t is discharge at time t.

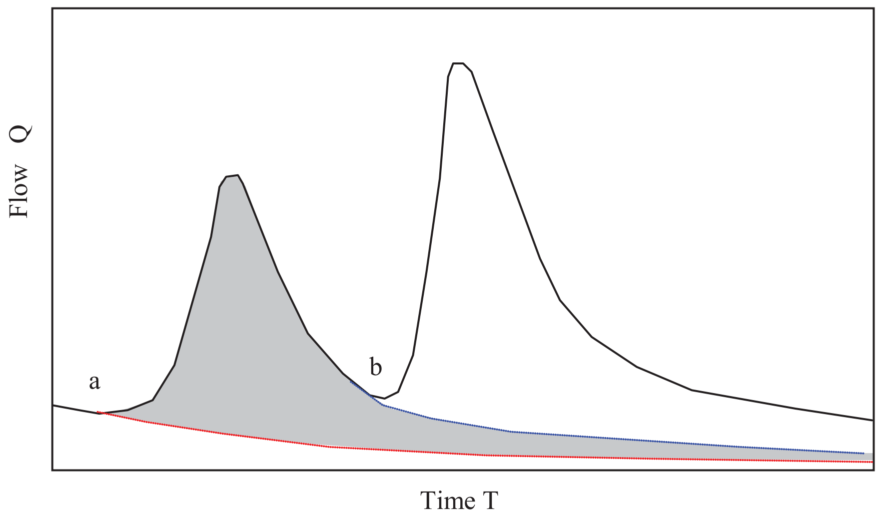


Figure 3 Flow event separation

Integrate this formula from $t = 0$ to $t = \infty$,

$$W_{ft} = \int_0^\infty Q_o e^{-t/k} dt = \left[-kQ_o e^{-t/k} \right]_0^\infty = kQ_o \tag{2}$$

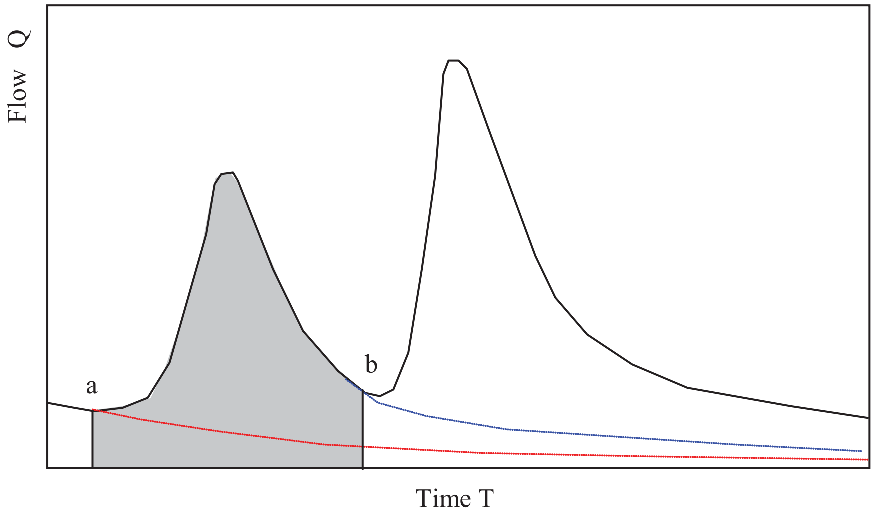


Figure 4 Flow event volume calculation

Thus, the total runoff volume (both direct runoff and base flow) for the shaded event can be calculated as

$$Runoff\ \ Volume = A_{shaded} + k\left(Q_b - Q_a\right) \tag{3}$$

6.3 Direct Runoff and Base Flow Separation

For each flow event derived from the above, we need to separate it into direct runoff and base flow. There is no standard means of differentiating between base flow and direct runoff. Generally, as time goes by from the peak of the flow, the dominance of surface and interflow will decrease and groundwater is gradually taking over. The N days in the diagram should be estimated from the actual catchment characteristics. For a rough guidance, an equation by Linsley (Linsley 1992, P45) could be used:

$$N = 0.8A^{0.2} \quad \text{(days)} \tag{4}$$

where A is catchment area in km^2.

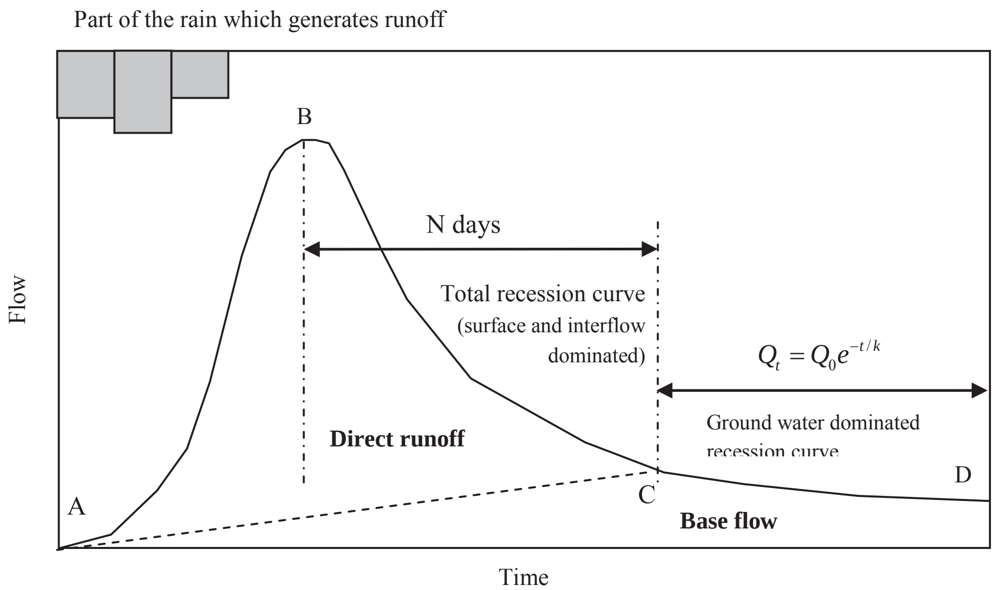
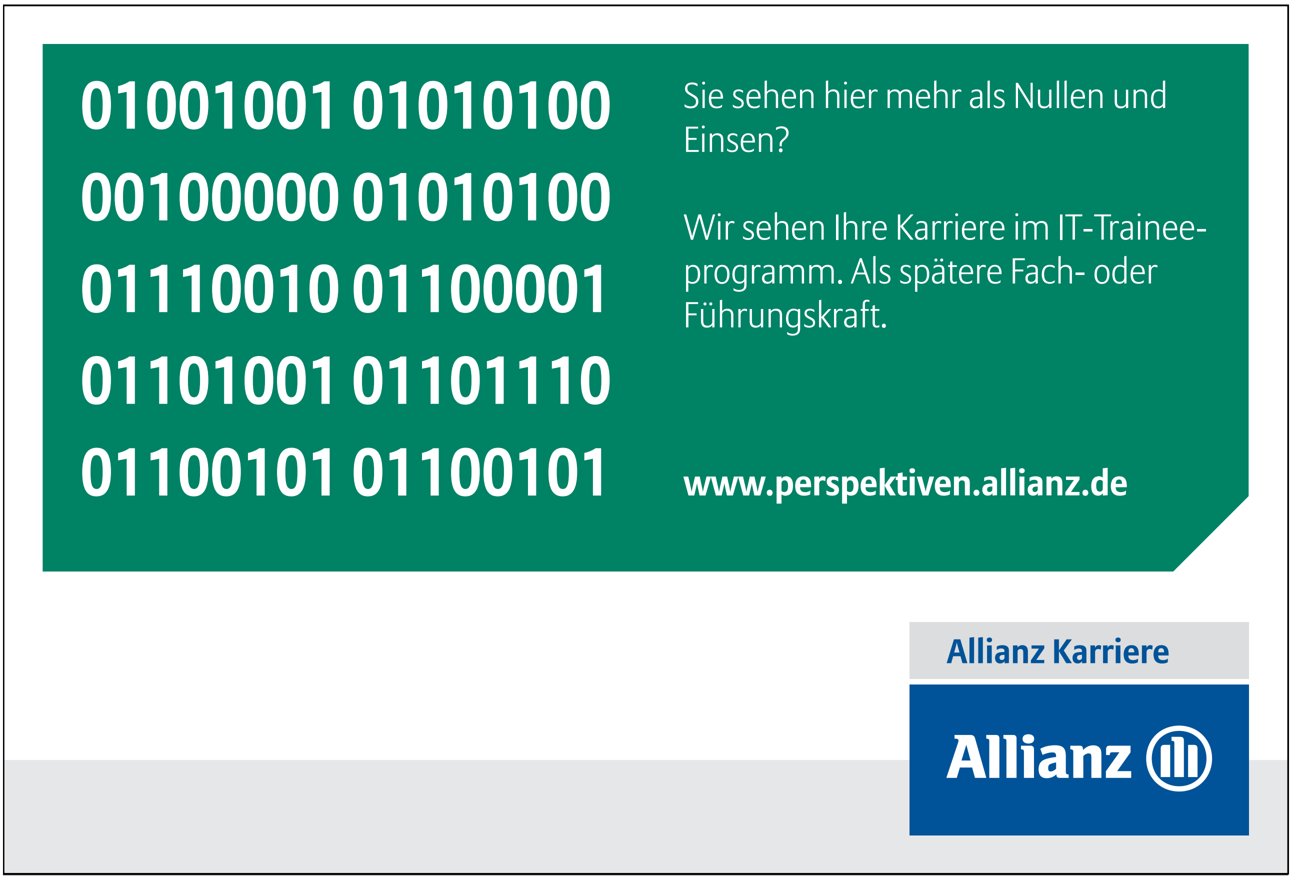


Figure 5 Direct runoff and base flow separation with an event hydrograph

The straight line A-C divides the flow into direct runoff and base flow.



In practice, it is also common that individual rainfall runoff events are not separated and base flow can be found by drawing a horizontal line (or a sloping line) along the event hydrographs (Figure 6).

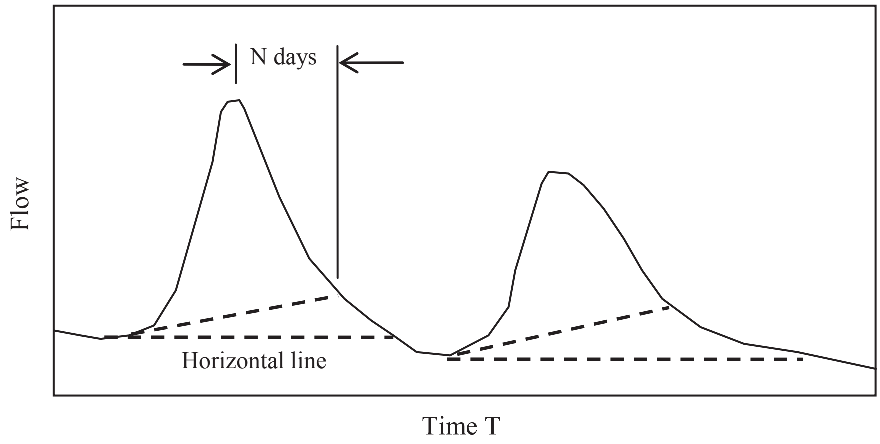


Figure 6 Direct runoff and base flow separation with the whole hydrograph

6.4 Effective Rainfall (Net Rainfall)

Effective rainfall (or net rainfall) is the part of the total rainfall that contributes to direct runoff. The difference between the total rainfall and the effective rainfall is called 'losses'. There are many ways to derive effective rainfall. The water balance equation is the foundation for these methods as shown below.

Total effective rainfall = Direct runoff volume (5)

6.4.1 The Φ index method

In this method, the rainfall losses are considered as constant with time (Figure 7).

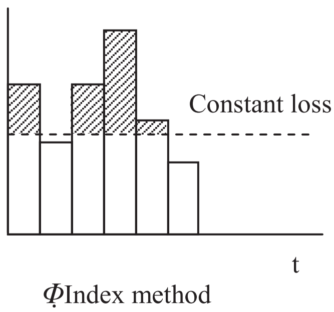


Figure 7 Φ index method

6.4.2 The initial and continuing losses

Some initial losses are attributed to interception from vegetation and land surface storage. The losses are then become constant after the initial losses.

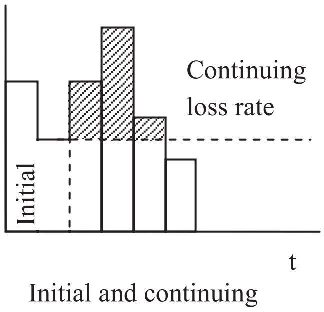


Figure 8 The initial and continuing losses

6.4.3 The proportional losses

It assumes that the same proportion of the total rainfall is lost to evapotranspiration, infiltration, etc.

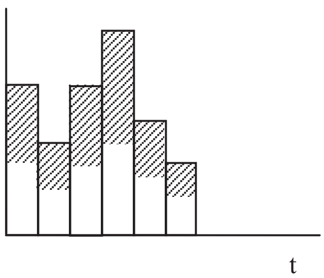


Figure 9 The proportional losses

6.4.4 Soil moisture accounting scheme

Although the aforementioned simple schemes are easy to use, they are too much simplified for actual effective rainfall processes and cannot provide the proportions of the effective rainfall related to surface runoff, interflow and base flow. Nowadays, soil moisture accounting models are increasingly used in effective rainfall estimation. This type of models includes several soil layers with the upper layer for the vegetation interception. For further information, readers can refer to HEC-HMS user manual (HEC, 2009). Other models such as Xinanjiang and PDM are also widely used.

6.5 Direct Runoff Modelling (Unit Hydrograph)

With the effective rainfall, a unit hydrograph model is usually used to model the direct runoff.

6.5.1 Unit hydrograph definition

The unit hydrograph is the catchment flow response to a unit (1 cm) of effective rainfall occurring over a given duration (Figure 10). It is assumed that a) effective rainfall is uniformly distributed over the whole catchment and the duration; b) direct runoff process is linear in superposition and proportionality (i.e., if rainfall is doubled, the runoff is also doubled. Any runoff from a later time can be added to the previous runoff); c) the rainfall runoff process is stationary (no change with time).

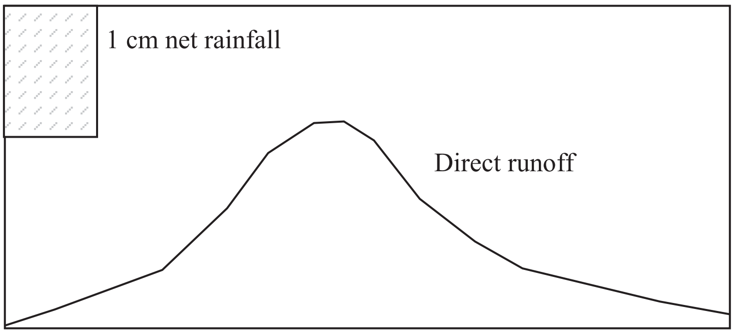


Figure 10 Unit hydrograph

The total flow from several rainfall values can be simply added together as shown in Figure 11.

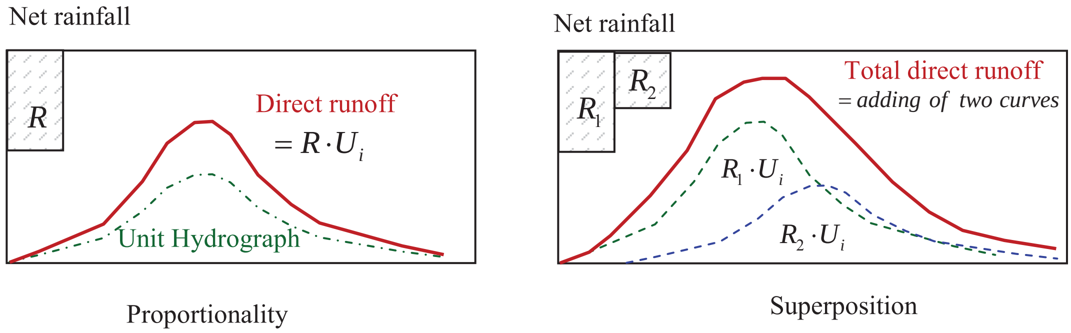


Figure 11 Proportionality and Superposition

6.5.2 Unit hydrograph application

The convolution formula for the unit hydrograph model is

$$Q_i = \sum_{m=1}^{n \le M} R_m U_{i-m+1} \tag{6}$$

where R_m is effective rainfall, U_i is unit hydrograph ordinates, Q_i is direct runoff, M is the number of rainfall values. In practice, it is much easier to use a table to carry out the calculations.

Practice

A river catchment has a 2 hour unit hydrograph with the ordinates 0, 3, 11, 35, 55, 66, 63, 40, 22, 9 and 2 m^3/s. Assume that the base flow at time t=0 hour is 20 m^3/s and linearly increases to 44 m^3/s at t=24 hours.

a) Compute the hydrograph resulting from two successive 2 hour periods of effective rain of 2.0cm and 1.5 cm respectively.

b) To prevent downstream flooding, the maximum flow to be released from the catchment is set at 180 m^3/s. Calculate the space needed to store the excess water from this event (in m^3).

Solution

Time(h)	UH(m^3/s)	R_1*U(t)	R_2*U(t-2)	Base flow	Total (m^3/s)	above 180m^3/s
0	0	0		20	20	
2	3	6	0	22	28	

4	11	22	4.5	24	50.5	
6	35	70	16.5	26	112.5	
8	55	110	52.5	28	190.5	10.5
10	66	132	82.5	30	244.5	64.5
12	63	126	99	32	257	77
14	40	80	94.5	34	208.5	28.5
16	22	44	60	36	140	
18	9	18	33	38	89	
20	2	4	13.5	40	57.5	
22	0	0	3	42	45	
24		0	0	44	44	
Rain R_1=	2	cm				Sum of the positive area (m^3)
Rain R_2=	1.5	cm				1299600

The results can be illustrated in Figure 12.

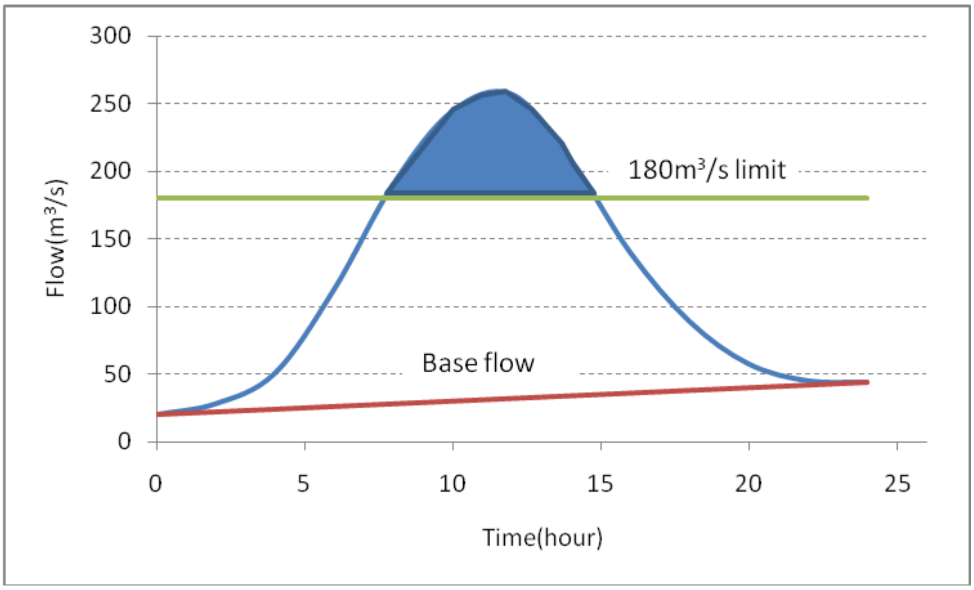


Figure 12 The event hydrograph

6.5.3 Unit hydrograph estimation

With the aid of computer programming, least squares can be used to estimate a unit hydrograph from the effective rainfall and direct runoff. First of all, a rainfall runoff event is to be selected from historical records. Effective rainfall and direct runoff can then be derived from the aforementioned methods. The linear system equation is described as

$$\mathbf{R} \cdot \mathbf{U} = \mathbf{Q} \tag{7}$$

where

$$\mathbf{R} = \begin{bmatrix} R_1 & 0 & \cdots & 0 & 0 \\ R_2 & R_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & R_M & R_{M-1} \\ 0 & 0 & \cdots & 0 & R_M \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_{N-M} \\ U_{N-M+1} \end{bmatrix} \text{ and } \mathbf{Q} = \begin{bmatrix} Q_1 \\ \vdots \\ Q_M \\ \vdots \\ Q_N \end{bmatrix}$$

It is very tedious to solve this linear system equation manually, but with MATLAB, Eq(7) can be easily solved with just one command line.

```
>>U= R\Q;     % solve the linear equation by QR decomposition
```

6.5.4 Unit hydrograph duration change (S-Curve)

To convert a unit hydrograph from one duration to another, an S-curve is used. An S-curve can be derived by assuming a continuous rain and adding up all the unit hydrograph ordinates (Figure 13).

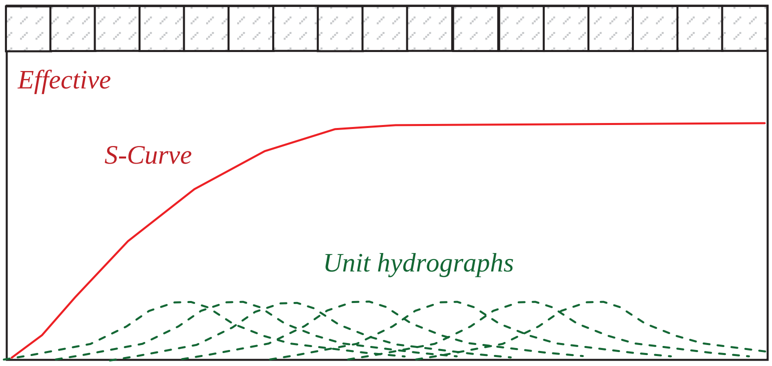


Figure 13 S-Curve derivation

A new unit hydrograph with a new duration Δt can be derived by shifting the S-curve to the new duration (such as half hour or two hours).

$$Q(\Delta t, t) = \frac{\Delta t_0}{\Delta t}\left[S(t) - S(t - \Delta t)\right] \tag{8}$$

where $Q(\Delta t, t)$ - the new unit hydrograph with duration Δt; Δt_0 - the duration of the original unit hydrograph; Δt - the new duration; $S(t)$ is S-curve; $S(t-\Delta t)$ is S-curve shifted by Δt. Interpolation of the S-curve is needed if a shorter duration unit hydrograph is to be estimated.

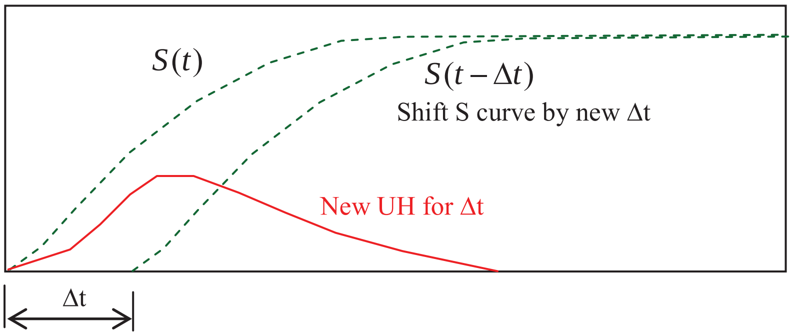


Figure 14 Unit hydrograph duration change

6.5.5 Synthetic unit hydrograph

The unit hydrograph estimated from rainfall runoff records of a specific catchment can only be applied to that catchment. However, many catchments are ungauged and there are no rainfall or runoff data to develop a unit hydrograph model. In such cases, a synthetic unit hydrograph can be estimated from a set of equations derived from the regression analysis of the gauged catchments. For example, a set of equations derived by Espey et al. from 41 catchments in the USA (1977, from Chow 1988 pp227) are as follows

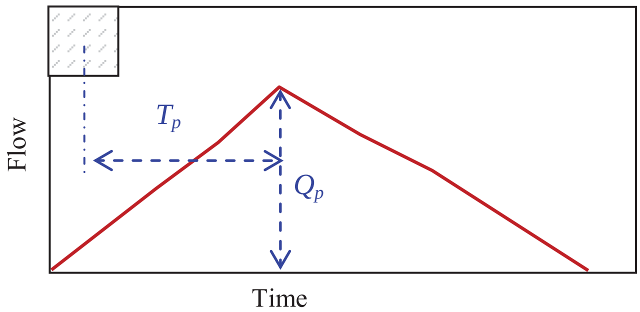


Figure 15 A synthetic unit hydrograph

$$T_p = 3.1L^{0.23}S^{-0.25}I^{-0.18}\Phi^{1.57} \quad \text{and} \quad Q_p = 31.62\times10^3 A^{0.96}T_p^{-1.07} \tag{9}$$

where L is the main river channel length, S is the main channel slope, I is the percentage of impervious area, Φ is linked with channel roughness and impervious area, A is catchment area.

Questions 6
Hydrograph

1. Use diagrams to show how to separate flow events and then divide the event flow hydrograph into direct runoff and base flow.
2. What are the assumptions in unit hydrograph model?
3. A river catchment has a 2 hour unit hydrograph with the ordinates 0, 3, 11, 35, 55, 66, 63, 40, 22, 9 and 2 m^3/s. Assume that the base flow at time t=0 hours is 50 m^3/s and linearly increases to 74 m^3/s at t=24 hours.

    a) Compute the hydrograph resulting from two successive 2 hour periods of effective rain of 2.0cm and 3.0 cm respectively.

    (Answers: 50, 58, 85, 159, 273, 357, 386, 333, 230, 152, 101, 78, 74 m^3/s)

    b) To prevent downstream flooding, the maximum flow to be released from the catchment is set at 273 m^3/s. Calculate the space needed to store the excess water in this event (in m^3).

    (Answer: around 1.85 million m^3)

4. Derive 2 hour 1cm Unit hydrograph from the following S-curve.

Time (hour)	0	1	2	3	4	5	6	7	8…
S(t) (m^3/s)	0	16	226	301	341	361	371	376	376

(Answers: 0, 8, 113, 142.5, 57.5, 30, 15, 7.5, 2.5, 0 m^3/s)

Solutions 6

Hydrograph

3. The computation is listed below

Time(h)	UH(m^3/s)	R_1*UH(t)	R_2*UH(t-2)	Base flow	Total (m^3/s)	above 273m^3/s
0	0	0		50	50	-223
2	3	6	0	52	58	-215
4	11	22	9	54	85	-188
6	35	70	33	56	159	-114
8	55	110	105	58	273	0
10	66	132	165	60	357	84
12	63	126	198	62	386	113
14	40	80	189	64	333	60
16	22	44	120	66	230	-43
18	9	18	66	68	152	-121
20	2	4	27	70	101	-172
22	0	0	6	72	78	-195
24		0	0	74	74	-199
Rain R_1=	2	cm				Sum of the positive area (m^3)
Rain R_2=	3	cm				1850400

4. From $Q(\Delta t, t) = \frac{\Delta t_0}{\Delta t}\left[S(t) - S(t-\Delta t)\right]$, and $\Delta t = 2$ hour, $\Delta t_0 = 1$ hour

Time (hour)	S(t) (m^3/s)	S(t-2)	UH(2hour)
0	0		0
1	16		8
2	226	0	113
3	301	16	142.5
4	341	226	57.5
5	361	301	30
6	371	341	15
7	376	361	7.5
8	376	371	2.5
	376	376	0

7. Flow Routing

Flow routing is a procedure to estimate downstream hydrograph from upstream hydrograph (Figure 1). Since flow routing has been widely used in flood estimations, flow routing is usually called flood routing. The routed hydrograph is delayed by a time lag (translation) and is attenuated. Flow routing is divided into river flow routing and reservoir flow routing.

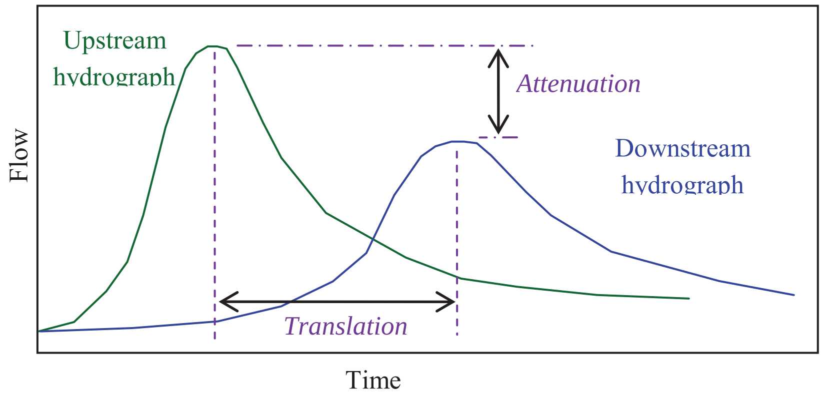


Figure 1 Flow routing

7.1 Basic Equations

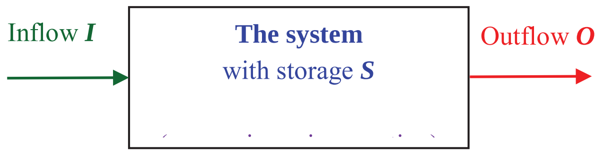


Figure 2 Flow routing system

From the conservation of mass, water balance for a system in Figure 2 can be expressed as

$$I - O = \frac{dS}{dt} \tag{1}$$

where I is upstream inflow , O is downstream outflow, S is the storage (reservoir or a river reach).

In practical calculation, it is more convenient to use a finite difference form of Eq(1) for a Δt duration. The mean values for the inflow and outflow are used instead of the instantaneous value.

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t} \tag{2}$$

To estimate the downstream outflow, it is also necessary to get the storage function that links the input and output.

$$S = f(I, O) \tag{3}$$

It is then possible to solve the outflow from Eq (2) and (3).

7.2 River Flow Routing (The Muskingum Method)

7.2.1 The outflow equation

The storage function in a river reach is linked with both inflow and outflow.

$$S = K\left[XI + (1 - X)O\right] \tag{4}$$

where K is the storage time constant for the reach, X is a weighing factor (between 0~ 0.5, usually around 0.2).

From the water balance equation $\frac{I_1+I_2}{2}-\frac{O_1+O_2}{2}=\frac{S_2-S_1}{\Delta t}$

So $\frac{I_1+I_2}{2}-\frac{O_1+O_2}{2}=\frac{K\left[XI_2+(1-X)O_2\right]-K\left[XI_1+(1-X)O_1\right]}{\Delta t}$

Simplify it to get the Muskingum equation

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 \qquad (5)$$

where $C_0=\left(0.5\Delta t-KX\right)/D$, $C_1=\left(KX+0.5\Delta t\right)/D$, $C_2=\left(K-KX-0.5\Delta t\right)/D$

$$D = K - KX + 0.5\Delta t$$

It is important to check if $C_0 + C_1 + C_2 = 1$. If not, some adjustments to the parameters are needed. If there are rounding errors, adjust the largest *C* value first.

Since I_1, I_2 and O_1 are known for every time step, O_2 is solved for successive time steps using each O_2 as O_1 for the next time step. O_1 is assumed the same as I_1 at the beginning if not given.

Practice

Estimate the downstream hydrograph using the Muskingum method with K=3hr and X=0.3. The time interval is 3 hours. The upstream hydrograph is as follows

Time (hr)	0	3	6	9	12	15	18
I (m^3/s)	1	3	9	15	13	10	6

Solution

Calculate the basic parameters

$$D = 3-3\times 0.3+0.5\times 3 = 3.6$$

$$C_0 = (0.5\times 3-3\times 0.3)/3.6 = 0.17$$

$$C_1 = (3\times 0.3+0.5\times 3)/3.6 = 0.67$$

$$C_2 = (3-3\times 0.3-0.5\times 3)/3.6 = 0.17$$

Check if $C_0 + C_1 + C_2 = 1$

$0.17+0.67+0.17=1.01$ let $C_1 = 0.66$ (change the largest weight)

With the routing equation $O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 = 0.17 I_2 + 0.66 I_1 + 0.17 O_1$

Time (h)	0	3	6	9	12	15	18
I (m^3/s)	1	3	9	15	13	10	6
O (m^3/s)	1	1.3	3.7	9.1	13.7	12.6	9.8

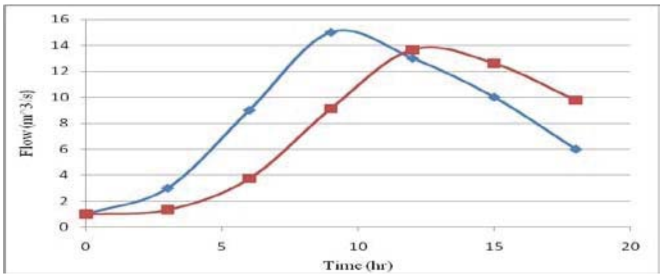


Figure 3 Muskingum routing results

7.2.2 Estimation of K and X

If there are no downstream records, the K value is estimated from the travel time in the reach (based on river bed slope and cross section). The X value is usually assumed as 0.2. If there are measured flow records downstream, more accurate X and K values can be derived from the procedures below.

From the Eq (4), $S = K\left[XI + (1-X)O\right]$, so S and $\left[XI + (1-X)O\right]$ have a linear relationship and its slope is K. The storage S can be worked out by accumulating $(I_{mean} - O_{mean})$ from each time step $S_t = \sum_{i=1}^{t} \overline{I}_i - \overline{O}_i$ (Figure 4). For $\left[XI + (1-X)O\right]$ calculations, only use the instantaneous values.

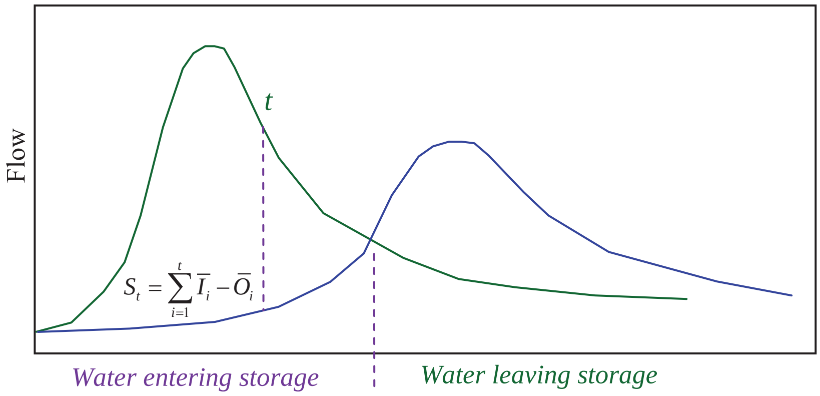


Figure 4 Storage calculation

Table 1 The K and X estimation table

Time (hr)	I (m^3/s)	O (m^3/s)	S	X=0.2	X=0.3	X=0.25
0	31	31	0	31.0	31.0	31.0
6	50	27	11.5	31.6	33.9	32.8
12	86	25	53.5	37.2	43.3	40.3
18	123	30	130.5	48.6	57.9	53.3
24	145	44	227.5	64.2	74.3	69.3
30	150	63	321.5	80.4	89.1	84.8
36	144	82	396	94.4	100.6	97.5
42	128	97	442.5	103.2	106.3	104.8
48	113	106	461.5	107.4	108.1	107.8
54	95	111	457	107.8	106.2	107.0
60	79	111	433	104.6	101.4	103.0
66	65	108	395.5	99.4	95.1	97.3
72	55	101	351	91.8	87.2	89.5
78	46	94	304	84.4	79.6	82.0
84	40	85	257.5	76.0	71.5	73.8
90	35	77	214	68.6	64.4	66.5

96	31	70	173.5	62.2	58.3	60.3
102	27	63	136	55.8	52.2	54.0
108	25	56	102.5	49.8	46.7	48.3
114	24	50	74	44.8	42.2	43.5
120	23	45	50	40.6	38.4	39.5
126	22	41	29.5	37.2	35.3	36.3

The results from Table 1 are plotted in Figure 5.

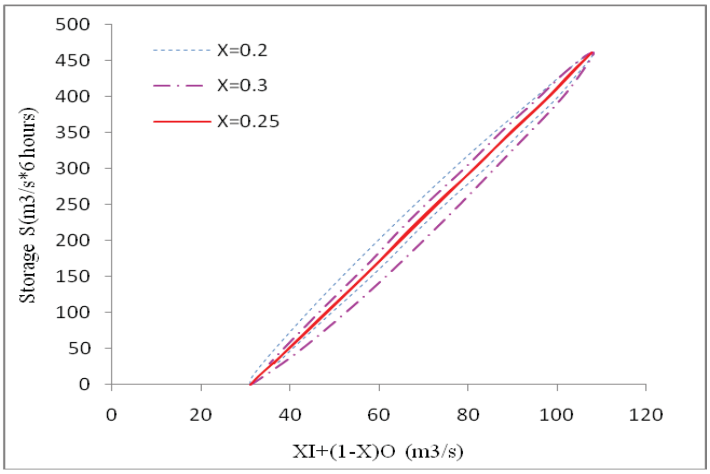


Figure 5 Storage loop diagram

From Figure 5, it is clear that *X*=0.25 curve has the narrowest loop, hence *X*=0.25 is chosen. The slope of the line is 6. Since the time interval is 6 hours, so $K = 6 \times 6hr = 36hr$. The line's intercept to the horizontal axis is not zero because the storage initial value is set as zero instead of 31. Since we are only interested in the slope, this is not a problem.

7.3 Reservoir Flow Routing

For a flood going through a reservoir, the water level in the reservoir is assumed as horizontal. The storage function would be linked with the reservoir water level.

$$S = f(h) \tag{6}$$

This function can be found from the topographic map. Since the storage below the spillway crest plays no role in the flow routing process, only the storage above the crest is considered.

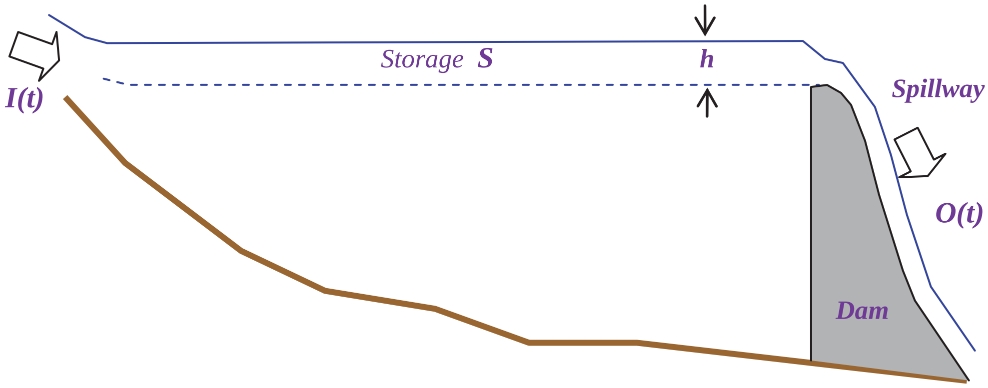


Figure 6 Reservoir flood routing

The discharge over the spillway crest is a function of h as well.

$$O = Cbh^{1.5} \tag{7}$$

where C is the discharge coefficient, b is the width of the spillway crest.

From Eq(2)

$$I_1 + I_2 - O_1 + \frac{2S_1}{\Delta t} = \frac{2S_2}{\Delta t} + O_2 \tag{8}$$

The knowns are on the left side and the unknowns are on the right side. The computation time interval is usually taken as

$$\Delta t \approx \frac{\text{Duration of the inflow rising limb}}{5} \tag{9}$$

Since Eq(8) is nonlinear, the outflow can be solved either by a graph method or MATAB. To use the graph method, a curve from the following equation is plotted as in Figure 7.

$$RS = \frac{2S}{\Delta t} + O \tag{10}$$

For each time step, Eq (8) is used to derive RS, and then the outflow O can be found from the curve if Figure 7.

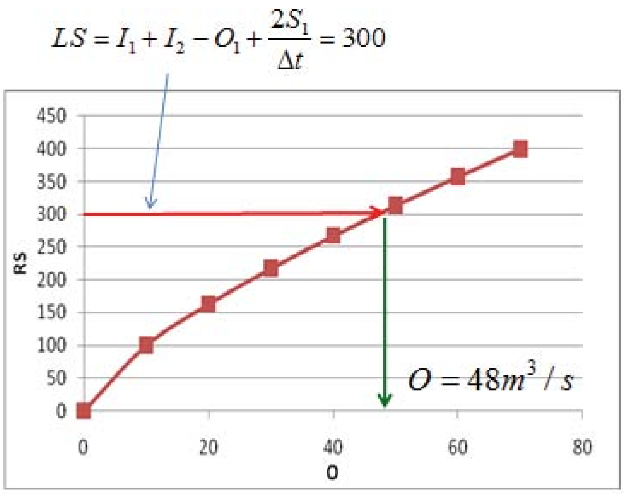


Figure 7 Curve RS versus O

A typical reservoir flow routing is illustrated in Figure 8. The outflow peak is attenuated by the reservoir, hence the downstrean flooding risk is reduced.

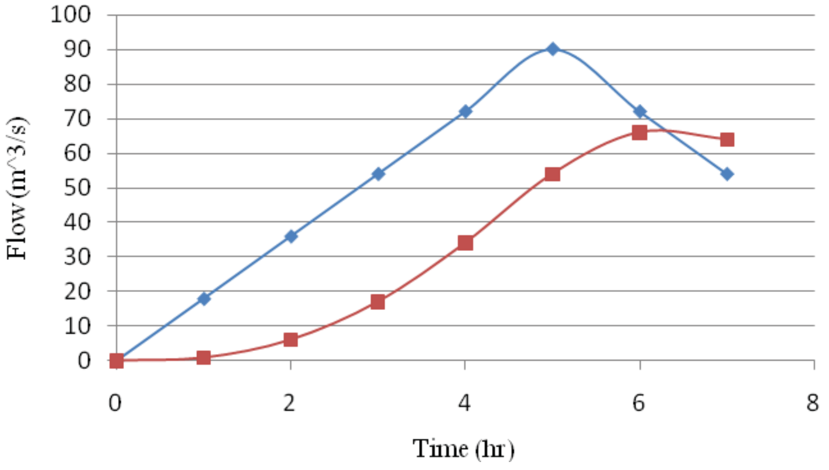


Figure 8 Reservoir flow routing result

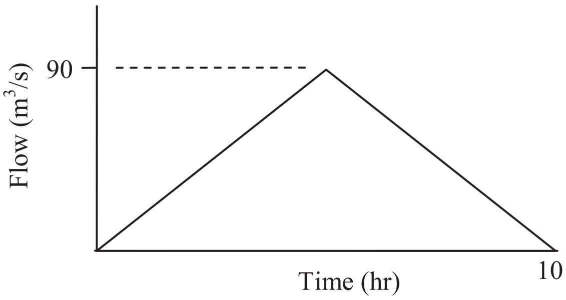
Questions 7
Flow Routing

1. The Muskingum method of flood routing has been chosen to forecast the movement of a flood wave from a point 30 km upstream of a centre of population. Engineers have previously estimated the Muskingum K and X parameters for this reach to be 10 hours and 0.15 respectively.

   If the upstream flood was measured every 6 hours starting at 0900 hours and the flows were recorded as 25, 35, 50, 80, 140, 130, 90, 80, 50, 30 and 25 m^3/s for the first 60 hours and 25 m^3/s thereafter then estimate the peak flow at the town and the time at which the peak flow will occur.

   (Answers: 113m^3/s and 36 hours later)

2. A triangular-shaped inflow hydrograph is routed through a reservoir assuming it is completely full at the beginning of the storm. The spillway crest is 20 m wide and has a coefficient of 2.7. The reservoir area is 0.5 km^2 and has vertical sides. What are the maximum outflow and the maximum height of water in the reservoir during this flow event?



(Answers: 66m^3/s, 1.14m)

Solutions 7

Flow Routing

1. Calculate the basic parameters

$$D = K - KX + 0.5\Delta t = 10 - 10 \times 0.15 + 0.5 \times 6 = 11.5$$

$$C_0 = \left(0.5\Delta t - KX\right) / D = (0.5 \times 6 - 10 \times 0.15) / 11.5 = 0.13$$
$$C_1 = \left(KX + 0.5\Delta t\right) / D = (10 \times 0.15 + 0.5 \times 6) / 11.5 = 0.39$$
$$C_2 = \left(K - KX - 0.5\Delta t\right) / D = (10 - 10 \times 0.15 - 0.5 \times 6) / 11.5 = 0.48$$

Check if $C_0 + C_1 + C_2 = 1$

Since $0.13 + 0.39 + 0.48 = 1$, it is correct.

With the routing equation $O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$
So
$$O_2 = 0.13 I_2 + 0.39 I_1 + 0.48 O_1$$

Time	0	6	12	18	24	30	36	42	48	54	60
I	25	35	50	80	140	130	90	80	50	30	25
O	25	26	33	46	71	106	113	100	86	64	46

The peak flow is 113m^3/s and occurs 36 hours later.

2. The computation time interval uses (duration of the inflow rising limb)/5=1 hour.

Hence

$\Delta t = 1$ (hr)=3600 (s)

$$O = Cbh^{1.5} = 2.7 \times 20h^{1.5} = 54h^{1.5}$$

Hence $h = (O / 54)^{2/3}$

The storage function is

$S = 500000h$

Combine the discharge and storage functions

$S = 500000(O/54)^{2/3} = 34951O^{0.667}$

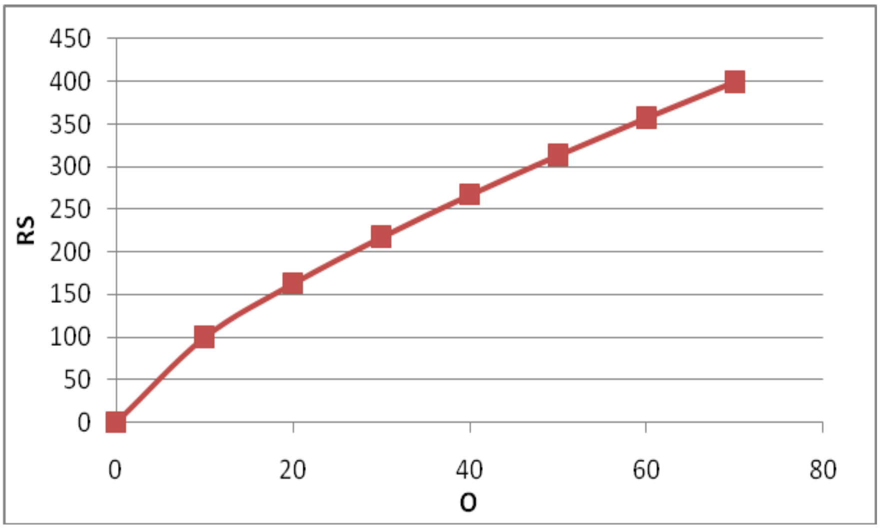
From the water balance $\dfrac{I_1 + I_2}{2} - \dfrac{O_1 + O_2}{2} = \dfrac{S_2 - S_1}{\Delta t}$

So $I_1 + I_2 - O_1 - O_2 = 19.4O_2^{0.667} - 19.4O_1^{0.667}$

Rearrange

$I_1 + I_2 - O_1 + 19.4O_1^{0.667} = O_2 + 19.4O_2^{0.667}$

A graph for the right side (RS=$O_2 + 19.4 O_2^{0.667}$) is created



The inflow values are as below.

Time	0	1	2	3	4	5	6	7	8	9	10
I	0	18	36	54	72	90	72	54	36	18	0
O	0										

Use $RS = LS = I_1 + I_2 - O_1 + 19.4 O_1^{0.667}$ to work out the RS one by one, check the graph to get the outflow (again, one by one) . Stop the calculation when the peak is reached. The following results are derived from MATLAB. If you derive them from the graph, the results would be slightly different.

Time	0	1	2	3	4	5	6	7	8	9	10
I	0	18	36	54	72	90	72	54	36	18	0
RS		18	70	148	239	332	385	377			
O	0	0.8	6.0	17	34	54	66	64			

The peak outflow is 66m^3/s. the maximum water height is $h = (O/54)^{2/3} = (66/54)^{2/3} = 1.14m$

8. Hydrological Measurements

Hydrological measurements are used to obtain data on hydrological processes. Academic research and practical engineering projects all depend on the hydrological data to calibrate and validate the relevant models.

8.1 Basic terms

Hydrological processes vary in time and space. Although they are continuous in time and space, they are usually measured at point samples. The following information is relevant to hydrological measurements.

8.1.1 Time series

A time series is a sequence of data points, measured typically at successive times, spaced at (often uniform) time intervals. For example, the rainfall measured by a rain gauge at a specific location is a time series.

8.1.2 Time domain

Time domain refers to the analysis of hydrological time series with respect to time. A time domain graph shows how a hydrological process changes over time. It uses tools such as auto-correlation and cross-correlation analysis.

8.1.3 Frequency domain

A frequency domain graph shows how much of the time series lies within each given frequency band over a range of frequencies. The frequency tools include spectral analysis and wavelet analysis.

8.1.4 Spatial data

Spatial data have some form of spatial or geographical reference that enables them to be located in two or three dimensional space (such as remote sensed images). Spatial data are often accessed, manipulated or analyzed through Geographic Information Systems (GIS).

8.1.5 Spatial time series

It is a collection of time series with spatial or geographical references. For example, the data from a network of rain gauges are a typical spatial time series.

8.1.6 Aliasing

It is an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled. In such a case, distortions will occur when the signal reconstructed from samples is different from the original continuous signal.

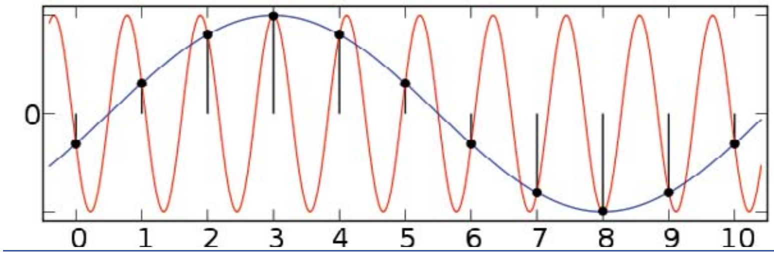


Figure 1 Signal aliasing (wikipedia 'aliasing')

8.1.7 Nyquist frequency

A perfect reconstruction of a signal is possible when the sampling frequency is greater than twice the maximum frequency of the signal being sampled. For example, if a signal has an upper band limit of 100 Hz, a sampling frequency greater than 200 Hz will avoid aliasing and allow theoretically perfect reconstruction. If a lower sampling rate has to be used, an anti-aliasing filter should be used to prevent aliasing.

8.2 Land based measurements

8.2.1 Rain gauge

Rainfall is recorded by two types of gauges: a nonrecording gauge is simply a container to store rain water. They are read manually at long time intervals (daily, weekly, etc.). In contrast, recording gauges automatically record the depth of rainfall with a high temporal resolution (available 15 minutes or hourly). A recording gauge has various ways of measuring rainfall intensity (tipping bucket, float, weighing, optical, etc.). The tipping bucket gauge is the most widely used rain gauge by the water industry due to its low cost and high reliability.

A rain gauge is more accurate than other rainfall measurement devices (e.g., weather radar and satellite), but it can only measure rainfall at a specific location, and its quality can be affected by wind, fallen tree leaves, etc.

Figure 2 A nonrecording gauge (left) and a recoding gauge (tipping bucket) used in the UK

8.2.2 Snow pillow

A snow pillow measures the water equivalent of the snow pack based on snow pressure on a plastic pillow.

8.2.3 Evaporation pan

An evaporation pan is used to hold water during observations for the determination of evaporation at a given location.

8.2.4 Lysimeter

A lysimeter is used to measure evapotranspiration and made with a tank of soil in which vegetation is planted to emulate the surrounding ground cover. The amount of evapotranspiration is measured by water weight balance from the water input and output of the tank.

8.2.5 River weir/flume

The discharge in a river (small to medium sizes) can be measured by a weir or flume (Figure 3). The water depth upstream of the weir/flume is measured and the discharge can be derived from the energy equation. For large rivers, water levels are measured and discharges are derived from the calibrated stage discharge rating curves.

Figure 3 A river weir in the River Brue, SW England

8.2.6 Soil moisture sensors

Soil moisture sensors measure water content in soil. There are three commonly used soil moisture sensors: capacitance sensor, tensiometer and neutron probe. All the sensors need to be calibrated for different soil types.

a) A tensiometer provides a direct measure of the tension at which water is held in soil. This instrument comprises a water filled tube which is sealed at one end, with a porous ceramic filter at the other end. When buried in soil, it allows water to flow freely through it, but not air. The suction of the water within the tube provides a direct measure of the suction pressure in the surrounding soil. With the suction pressure and soil moisture content curve, soil moisture can then be derived.

b) A capacitance sensor uses capacitance to measure the soil water content. It is a simple sensor made from two plates and the capacitance between them is measured to derive the soil water content.

c) A neutron moisture meter consists of two main components, a probe and a gauge. The probe is inserted in a hole in the ground and it emits fast neutrons. The emitted neutrons are slowed down and reflected by the water molecules in the surrounding soil. The gauge monitors the flux of the slow neutrons scattered by the soil. The degree of reflection is proportional to the soil moisture content. The operator of a neutron probe needs nuclear safety training.

8.2.7 Infiltrometer

Infiltrometer is a device used to measure the rate of water infiltration into soil or other porous media. Commonly used infiltrometers are of single ring or double rings. It is easy to use, but the soil structure is usually disturbed.

8.2.8 Radiation sensors

Solar radiation is shortwave and the earth's radiation is longwave (infrared) and they are measured by different devices.

Pyranometer (also called solarimeter) is used to measure solar radiation on a planar surface. The solar radiation is absorbed by a blackbody thermopile and the temperature difference between the metal in the radiation and the one under the shade represents the solar radiation intensity. A plastic dome is used to block longwave radiation so that only shortwave radiation is measured.

Pyrgeometer is a device that measures infrared radiation. Its working mechanism is similar to pyranometer except its plastic shield blocks shortwave radiation.

Net radiometer is used to measure net radiation at the earth's surface (incoming radiation minus outgoing radiation). Two radiation sensors (one upward facing and one downward facing) are needed to derive net radiation. If both net radiations for shortwave and longwave are needed, four sensors will be required.

A sunshine recorder is originally made with a glass sphere filled with water and later on with a solid glass sphere. When the sphere burns, it records a trace on the recorder cards attached to it, the length of which shows the duration of bright sunshine.

8.2.9 Anemometer

Anemometer is a weather instrument that measures wind speed. The most widely used anemometer consists of three or four cups that spin according to the speed of the wind. Modern ultrasonic anemometers are able to measure wind speed in three dimensions.

8.2.10 Air Temperature

Thermometers placed in a Stevenson screen are used to measure the ordinary, maximum/minimum air temperatures.

8.2.11 Hygrometer

Hygrometers are used for measuring relative humidity. Old style hygrometers with wet and dry bulb thermometers are called psychrometer. Evaporation from the wet bulb lowers the temperature, so that the wet-bulb thermometer usually shows a lower temperature than that of the dry-bulb thermometer. The dryer the air, the larger the temperature difference will be. A psychrometer depends on the accuracy of its thermometers. If one or both of the thermometers is off, large errors will occur. Modern electronic hygrometers use the changes in electrical resistance due to temperature condensation, and changes in electrical capacitance to measure humidity changes. Electronic hygrometers are extremely accurate and can continuously and automatically record relative humidity.

8.2.12 Barometer

A barometer is an instrument used to measure atmospheric pressure. There are various types based on air, water or mercury. Italian physicist Torricelli invented the first mercury barometer with a tube of 1m long filled with mercury. A more widely used barometer is called aneroid barometer which is made of a small, flexible alloy metal cell. Small changes in external air pressure cause the cell to expand or contract so that the attached mechanical levers can amplify the tiny movements of the capsule for visual display.

8.2.13 Weather radar

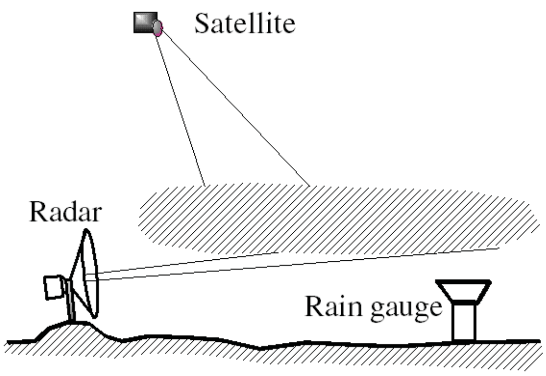


Figure 4 Weather radar, satellite and rain gauge

In contrast to a rain gauge that is a ground based measurement, weather radar measures rainfall well above the ground (Figure 4). Weather radars send directional pulses of microwave radiation. Between each pulse, the radar serves as a receiver and listens for return signals from rainfall drops in the air. Return echoes, called reflectivity, are analysed for their intensities in order to establish the precipitation rate in the scanned volume. Weather radars can cover large areas and are able to observe precipitation over the sea. Several radars can be combined to provide a composite rainfall image (Figure 5). There are several error sources in weather radar measurements. Radar pulses spread out as they move away from the radar station, decreasing resolution at far distances. Radar beams may also suffer from attenuation, shielding, anomalous propagation, brightband, etc.

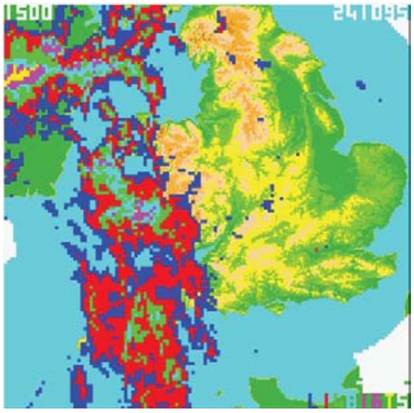


Figure 5 A composite weather radar rainfall image over the UK

8.3 Air based measurements

8.3.1Weather balloon

A radiosonde (Sonde is French for probe) is a unit for measuring various atmospheric parameters and transmits them to a fixed receiver. Modern radiosondes measure the following variables: pressure, temperature, relative humidity, wind speed and direction. A GPS with the radiosonde provides the location information (altitude, latitude/longitude). Worldwide there are more than 800 radiosonde launch sites. They are routinely launched at the same time twice a day (0000 and 1200 UTC) to provide an instantaneous snapshot of the atmosphere.

8.3.2 Aircraft

Aircrafts with various measuring devices are used to carry out a short period of intensive measurement over a study area. Lidar is a commonly used measuring device on an aircraft to derive high resolution digital terrain maps, especially during flood inundation events. Lidar (Light Detection And Ranging) is an optical remote sensing device that measures the elevations of ground and water surfaces using laser light beams.

8.4 Space based measurements

Satellites are increasingly used to collect meteorological and environmental variables that can be used in hydrological investigations. Two types of satellites are usually of interest to hydrologists and they are weather satellite and earth observation satellite. A weather satellite is primarily used to monitor the weather and climate of the earth such as clouds, precipitation, soil moisture, evapotranspiration, hurricane movement, etc. An earth observation satellite gathers information about the earth’s physical, chemical and biological systems such as land use, natural disasters (floods, land slide, forest fire, …), vegetation cover, land surface elevation, etc.

8.4.1 Orbit

Satellites can be either polar orbiting, seeing the same swath of the earth every 12 hours, or geostationary, hovering over the same spot on the earth by orbiting over the equator while moving at the speed of the earth's rotation.

A geostationary orbit (or Geostationary Earth Orbit - GEO) is a geosynchronous orbit directly above the earth's equator (0° latitude). The satellite orbits in the direction of the earth's rotation, at an altitude of 35,786 km above ground with an orbital period equal to the Earth's period of rotation. Geostationary orbits are useful because they cause a satellite to appear stationary with respect to a fixed point on the rotating earth. As a result, an antenna can point in a fixed direction and more frequent measurements could be made (e.g., at a sampling interval of 15 minutes or 30 minutes).

Polar orbiting weather satellites circle the earth at a typical altitude of 800 km in a north to south (or vice versa) path, passing over the poles in their continuous flight. Polar satellites are usually in sun-synchronous orbits, which means they are able to observe any place on the earth and will view every location twice each day with the same general lighting conditions due to the near-constant local solar time. Polar orbiting weather satellites offer a much better spatial resolution than their geostationary counterparts due to their closeness to the earth, but their temporal resolutions are much worse (a sampling interval of 1day or more).

8.4.2 Spectrum

The satellite remote sensing covers a wide range of electromagnetic spectrum. The most common are visible, infrared and microwave. Visible light can provide true colour images, but is only available during the day time. It is useful to spot clouds with visible light band (wavelength between 380 nm and 760 nm, i.e., 790–400 terahertz). Infrared images can be obtained during the daytime and night hence they are available around the clock, and they are useful in detecting object temperature values. Infrared covers a spectrum of 300 GHz (1 mm) to 400 THz (750 nm). Microwave covers as long as one meter to as short as one mm, or equivalently, with frequencies between 300MHz (0.3 GHz) and 300 GHz. Microwaves are absorbed by molecules that have a dipole moment in liquids. The commonly use microwave bands are L band from 1 to 2 GHz(15- 20 cm), S - Band 2 to 4 GHz (7.5-15 cm), C-band 4 to 8 GHz (3.75-7.5 cm), X band 8.0 to 12.0 GHz (2.5-3.75cm), Ku band 12 ~ 18 GHz (1.7-2.5 cm), K-band 18-27 GHz (1.1-1.7cm) and Ka band 26.5–40 GHz (0.75-1.1 cm). Short wavelength microwave is affected by more attenuation in the atmosphere and is usually used to measure atmospheric properties. Long wavelength microwave can penetrate the atmosphere with little attenuation and is used to measure ground surface properties such as soil moisture.

8.4.3 Passive and active microwave

There are two types of remote sensing devices. A passive device has receivers (called radiometer) that detect natural radiation emitted or reflected by the measured object. Reflected sunlight is the most common source of radiation measured by passive radiometers. An active device, on the other hand, has transmitters and receivers. A transmitter sends microwave beam to the target object and a receiver detects the radiation that is reflected or backscattered from the target. Radar is an example of active remote sensing where the time delay between emission and return is measured to find the distance between the target and the sensor.

8.4.4 Validation

It is not easy to validate satellite measurements due to the different footprints of the satellite (many sq km in size) and ground based measurement (e.g., rain gauge, a point measurement). Large number of ground based measurement points with long observation periods are needed for effective validations.

8.5 Transportable Weather Station

A weather station is a facility with instruments for observing weather related conditions. The measurements include air temperature, air pressure, humidity, wind speed, wind direction, solar radiation and precipitation. Wind measurements are taken as free of other obstructions as possible, while temperature and humidity measurements are kept free from direct solar radiation (housed in a Stephenson screen).

For some short term hydrological investigations, it is quite convenient to use a transportable weather station (TWS). The following instruments are commonly included in a TWS: 1) anemometer (wind speed m/s), 2) solarimeter (solar radiation kW/m^2), 3) net radiometer (kW/m^2), 4) psychrometer (wet and dry bulb temperatures ^{o}C), 5) precipitation detector (1 = wet; 0 = dry), 6) rain gauge (rainfall depth mm), 7) barometer (pressure in mb), 8) wind direction sensor (degrees)

A TWS is equipped with the following accessories: electricity supply (12V mains adapter or 12V rechargeable internal battery), a solar panel to charge internal battery (to face south in order to receive as much direct sunlight as possible), onboard data logger storing data on memory cards (1 min data resolution can be stored for up to 2 weeks unattended), PC serial connector for data download or transmission via a communication link.

Questions 8

Hydrological Measurements

1. Use Wikipedia and Google/Yahoo to explore the concepts and instruments introduced in this note.

2. What is aliasing? and how to avoid it?

3. How many types of sensors are available for measured soil moisture content? Explain their working mechanisms.

4. Rainfall can be measured either by a network of rain gauges or by using a remote sensing device such as weather radar. Briefly discuss the advantages and disadvantages of these two approaches.

5. Transportable Weather Stations (TWS) are frequently used in hydrology for relatively short-term measurement of key meteorological data at sites of interest. Describe a typical TWS and outline its principal instrumentation and data collection system. Use sketches where appropriate in describing individual sensors.

9. Hydrological Statistics

Hydrological processes are driven by physical, chemical and biological principles, the so called 'Laws of Nature'. However, in real life, the hydrological setting is usually of such complexity that the underlying hydrological processes cannot be modelled on first principles. Therefore, statistical models may be needed to link the hydrological processes in a descriptive way instead of a cause-effect relationship. Probability is the foundation for statistics and some relevant probability concepts for flood risk management are introduced here.

9.1 Basic Terms

9.1.1 Probability

Probability is a measure of how likely that some event will occur.

If a random event occurs a large number of times $\boldsymbol{n}$ and the event has attribute $\boldsymbol{A}$ in $\boldsymbol{n}_a$ of these occurrences, then the probability of the occurrence of the event having attribute $\boldsymbol{A}$ is

$$P(A) = \lim_{n\to\infty} \frac{n_a}{n} \approx \frac{n_a}{n} \tag{1}$$

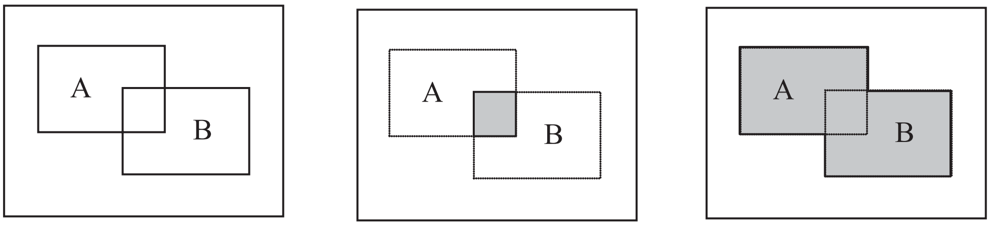
For this probability estimate (based on relative frequency) to be very accurate, n may have to be quite large. Since it is impossible to get an infinite number of observations, the actual probability in hydrology can only be approximated.

9.1.2 Return Period

Return period is the average time interval between occurrences of a hydrological event of a given or greater magnitude, usually expressed in years. For example, a 100-year flood will occur on average once in every 100 years. It is the inverse of the probability of an event. Therefore, a 100 year flood has 1% probability to occur in each year.

If a place has a 2% (0.02) probability of a flood striking in any given year, then that community would expect such a flood, on average, every 50 years (1/0.02).

9.1.3 Probability relationships



$A \cap B$ $A \cup B$

Figure 1 Venn diagram showing $A \cap B$ and $A \cup B$

Union and Intersection

$\cap$ - intersection (join probability), so $A \cap B$ represents all elements simultaneously in both *A* and *B*.

$\cup$ - union, so $A \cup B$ represents all in *A* or *B* or both.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{2}$$

If A and B are exclusive

$$P(A \cup B) = P(A) + P(B) \tag{3}$$

The conditional probability of B given A

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \quad \text{so} \quad P(A \cap B) = P(A)P(B \mid A) \tag{4}$$

If A and B are independent

$$P(B \mid A) = P(B), \quad \text{so} \quad P(A \cap B) = P(A)P(B) \tag{5}$$

Example 1
In a Bristol river, an annual peak flow in excess of 10 m^3/s has a return period of 100 years. If the annual peak flows are independent between the years, estimate the probability of such a peak flow will occur in 2 consecutive years.

Solution
From Eq (5)
The probability of the peak flow in excess of 10m^3/s is 0.01, so the probability of its occurrence in two consecutive years will be
$0.01 \times 0.01 = 10^{-4}$

Total probability *(or weighted probability)*

If B1, B2, …, Bn represents a set of mutually exclusive and collectively exhaustive events, one can determine the probability of another event A from

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i)P(B_i) \tag{6}$$

For example, the solar radiation analysis could be divided into rainy days and nonrainy days so that the total solar radiation could be derived.

9.1.4 Probability distributions

Discrete distribution

Bernoulli distribution: is a random process that can be either of two possible outcomes, ‘flooded’ and ‘no flooded’, ‘rainy’ and ‘non-rainy’, ‘heads’ or ‘tails’, etc.

$$f(x;p)=\begin{cases} p & \text{if x=1,} \\ 1-p & \text{if x=0.} \end{cases} \tag{7}$$

Binomial distribution: among n trials of Bernoulli process, the probability of x occurrence is

$$f(x;n,p)=\binom{n}{x}(p)^x(1-p)^{n-x} \quad x=0,1,2,...,n \qquad \text{and} \qquad \binom{n}{x}=\frac{n!}{(n-x)!x!} \tag{8}$$

Its expected value is

$$E(X)=np \tag{9}$$

Example 2

On average, how many times will a 10-year flood occur in a 40 year period? What is the probability that exactly this number of 10-year floods will occur in a 40 year period?

Solution

A 10-year flood has $p=1/10=0.1$

$E(X)=np=40(0.1)=4$

The probability of such a flood occurring 4 times in 40 years is

$$f(4;40,0.1)=\binom{40}{4}(0.1)^4(0.9)^{36}=0.2059$$

This problem illustrates the difficulty of explaining the concept of return period. On the average, a 10-year event occurs once every 10 years and 4 times in a 40 year period. Yet in about 80% (100(1-0.2059)) of the 10-year event will not occur exactly 4 times. As a matter of fact, the probability that it will occur 3 times is nearly identical to the probability it will occur 4 times (0.2003 vs 0.2059). The number of occurrences, X, is a truly random variable (with a binomial distribution).

Continuous distribution

Normal distribution (or Gaussian distribution): $X \sim N(\mu,\sigma^2)$ with the mean μ and variance σ^2

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{10}$$

There are many other probability distribution functions such as Gumbel, log normal, Pearson III, General Logistic, etc.

9.2 Statistical Flood Estimation

The statistical procedures in this section are used to derive flood peaks that are useful for designing flood defence projects.

9.2.1 Empirical probability

Empirical probability is a nonparametric probability (no theoretical distribution curves). To work out the empirical probability, rank the data points in descending order (from the largest to smallest). If n is the total number of data points and m is the rank of an individual point, the exceedance probability of the *mth* largest value, x_m is

$$P_m = P(X \geq x_m) = \frac{m}{n} \quad \text{(California formula)} \qquad (11)$$

$$P_m = P(X \geq x_m) = \frac{m}{n+1} \quad \text{(Weibull formula)} \qquad (12)$$

Eq(11) is easy to apply, but biased (over-estimated) and it is impossible to plot the nth data point (i.e., 100% probability) on most probability graph papers. For example, with a 100 year flood record, the largest flood is ranked 1, hence its exceedance P=1/100, i.e., 100 year return period. But the smallest one is ranked 100 and its exceedance P=100/100=1, which is unrealistic (it means any floods will be larger than this flood and no floods can be smaller). Eq(12) is able to overcome this problem and is the most popular formula among hydrologists despite it is only the best for uniform distributions. In FEH (Vol 3 pp143), the Gringorten formula is recommended for the flood distribution in the UK. For the problems in this unit, Eq(12) is recommended. For practical engineering problems, the relevant hydrological manuals should be consulted.

The return period T of the event $X \geq x_T$ and the probability are linked by

$$P(X \geq x_T) = \frac{1}{T} \tag{13}$$

9.2.2 General procedure for flood estimation

a) Obtain a maximum flood in each year in n years.

$x_1, x_2, x_3, \ldots x_n$

For example:

Year	1987	1988	1989	1990
Q	25.1	41.5	29.9	21.2

b) Rank the data from high to low

Year	Q	Rank
1987	25.1	3
1988	41.5	1
1989	29.9	2
1990	21.2	4

or

Year	Q	Rank
1988	41.5	1
1989	29.9	2
1987	25.1	3
1990	21.2	4

c) Use the empirical equation (12) to work out the plot positions for the exceedance probabilities (n=4 in this case).

Year	Q	Rank	P(%)
1988	41.5	1	20
1989	29.9	2	40
1987	25.1	3	60
1990	21.2	4	80

d) Plot the floods and exceedance probabilities on a probability paper. In this exercise, log probability paper is used. For other types, seethe link in the reference list for Graph paper 2009. The unit labels on the vertical axis can be scaled to fit the actual data. In this case, each tick represents 10 m^3/s.

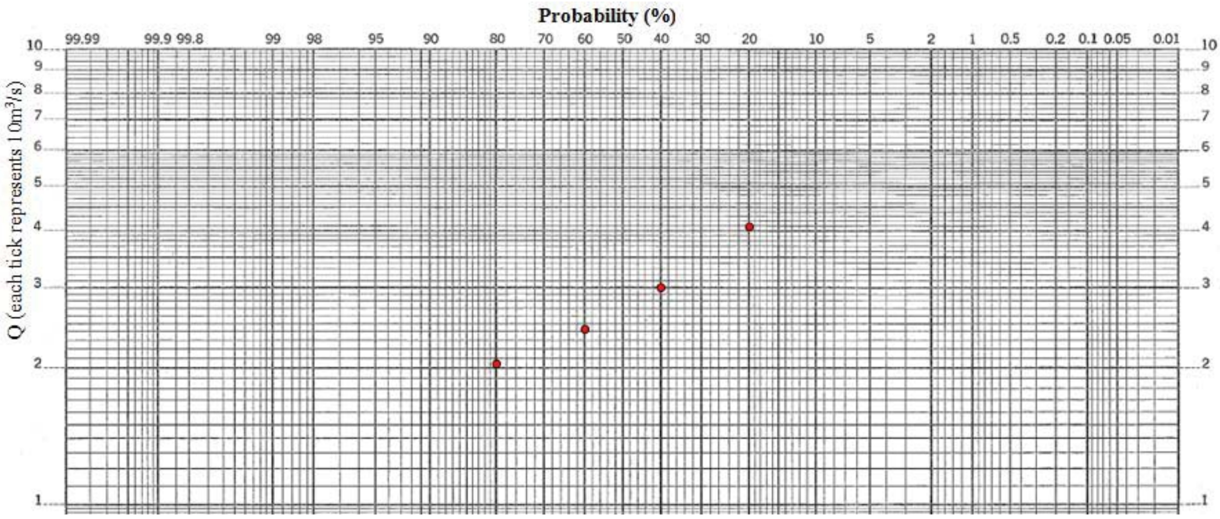


Figure 2 Plot on Log Probability paper

e)

Draw a straight line among the points to best fit the data points.

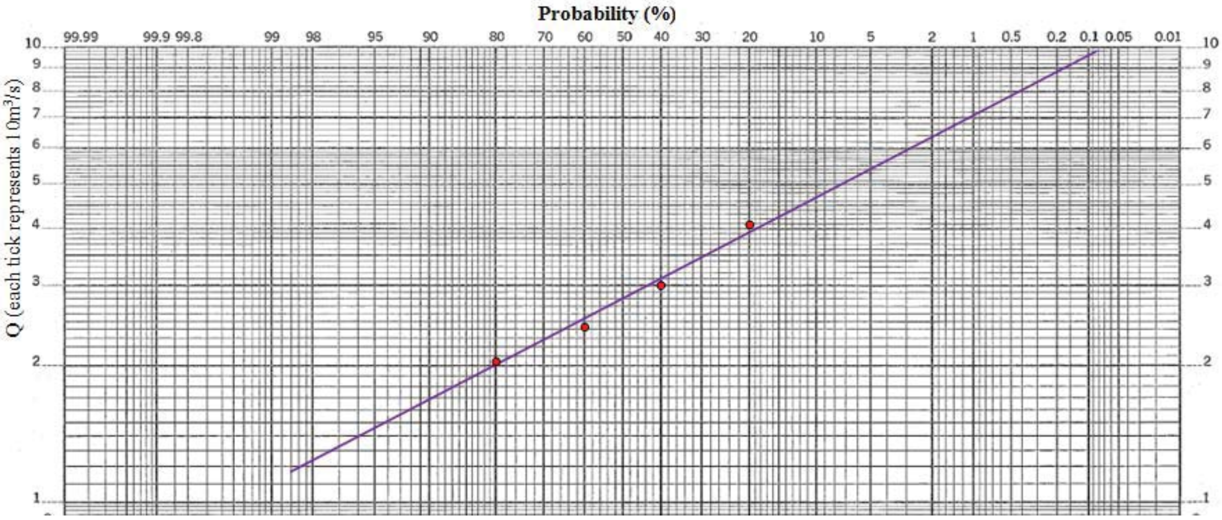


Figure 3 Fit a line to the data points

f) The corresponding floods and return period (probabilities) can be read from the line.

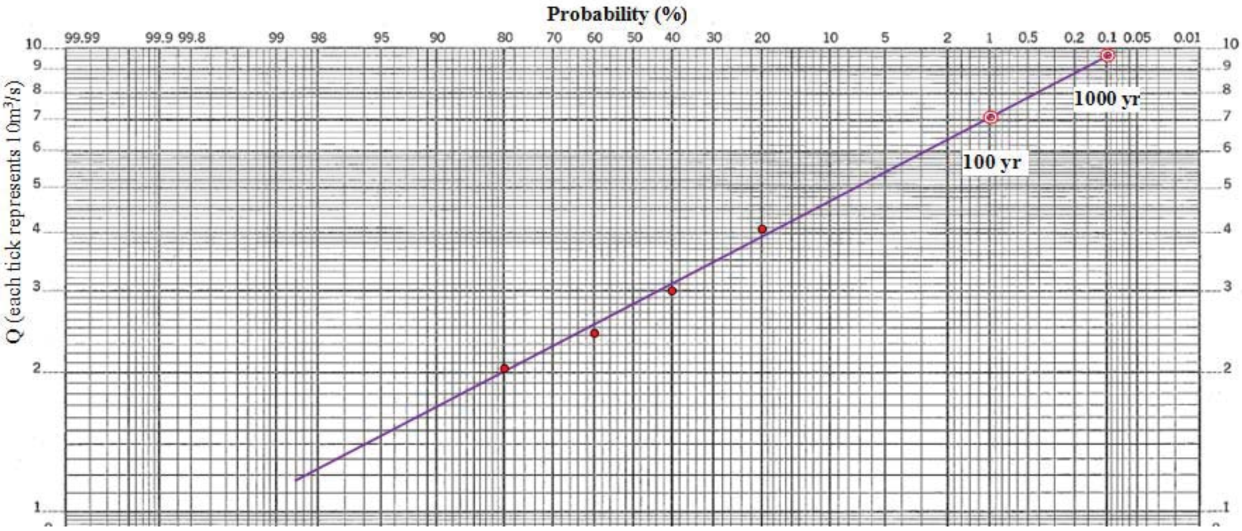


Figure 4 Floods with 100 year and 1000 year return period can be read from the curve.

It should be noted that the estimations for 100 and 1000 year floods based on only 4 data points are very unreliable and more data points should be used in practical projects. The 4 data points here are used for illustrations only.

9.3 Statistical Rainfall Estimation

The rainfall frequency procedures have two purposes: *the estimation of design rainfall depths, and the assessment of the rarity of observed rainfall events.* Design rainfalls are required principally for river flood estimation, here they are an important component in the design for flood defences, bridges, culverts and reservoir spillways. Many flood estimates depend on good rainfall frequency information because rainfall records tend to be more plentiful and longer than river flow records. Other applications for design rainfalls can be found in agriculture and sewage design for built-up areas and drainage for buildings.

The rainfall statistics involve both rainfall depth (or intensity) and duration. It is a complex process to derive frequency curves from rainfall data. The useful diagram for rainfall statistics is called DDF (Depth Duration Frequency) or IDF (Intensity Duration Frequency). They can be converted from one to the other. An example DDF is shown in Figure 5.

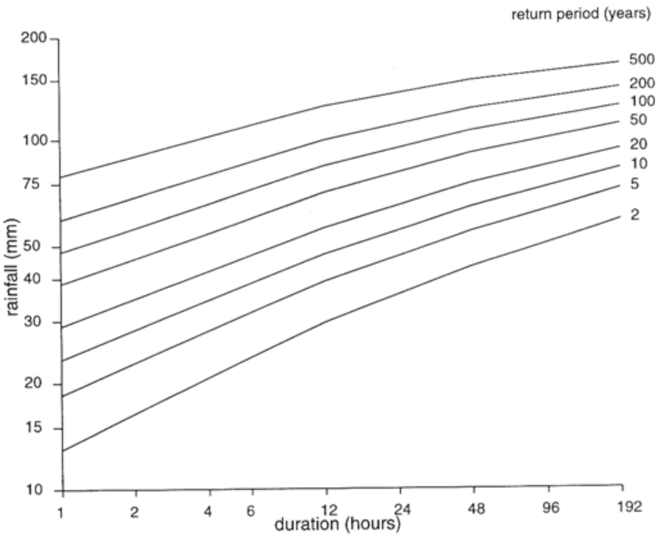


Figure 5 Depth-Duration-Frequency curves

The DDF diagram can be used to assess the rarity of observed rainfall events (Figure 6a) when the duration and rainfall depth information is known, or to estimate design rainfall with a predefined return period (Figure 6b).

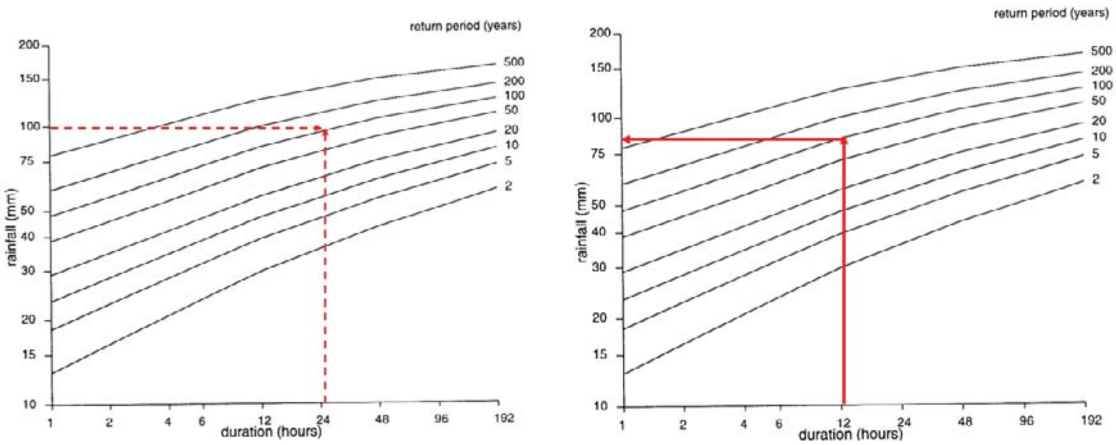


Figure 6 Application of Depth-Duration-Frequency curves a) rarity check, b) design rainfall

Questions 9

Hydrological Statistics

1. On average, how many times will a 10-year flood occur in a 40 year period? What is the probability that three 10-year floods will occur in a 40 year period? What is the probability that such a flood will not occur at all in a 40 year period? What is the probability that such a flood will occur at least once in a 40 year period? (Hint: use the Binomial distribution)

   (Answers: 4, 0.2003, 0.0148, 0.9852)

2. If the annual maximum flows for a catchment in England between 1987 and 1996 were 25.1, 41.5, 29.9, 21.2, 35.5, 23.8, 25.5, 28.0, 33.0 and 31.5 cumecs, estimate the 20, 50 and 100 year return period flows assuming that they were distributed in accordance with a log normal distribution (i.e., use a Log probability paper).

   (Download a sheet of Log 2 cycle probability paper at
   http://sorrel.humboldt.edu/~geology/courses/geology531/graph_paper_index.html)

   (Answers: 45, 50, 53 m^3/s)

Solutions 9

Hydrological Statistics

1) This is a Binomial distribution.

A 10-year flood has $p = 1/10 = 0.1$

$E(X) = np = 40(0.1) = 4$

The probability of such a flood occurring 3 times in 40 years is

$$f(3;40,0.1) = \binom{40}{3}(0.1)^3(0.9)^{37} = \frac{40!}{37!3!}(0.1)^3(0.9)^{37} = 0.2003$$

The probability of such a flood occurring 0 times in 40 years is

$$f(0;40,0.1) = \binom{40}{0}(0.1)^0(0.9)^{40} = \frac{40!}{40!0!}(0.1)^0(0.9)^{40} = 0.0148$$

Since all the probabilities should be added to 1, i.e.,
P(0)+P(1)+(2)+…=1 so P(1)+P(2)+… = 1 - P(0)

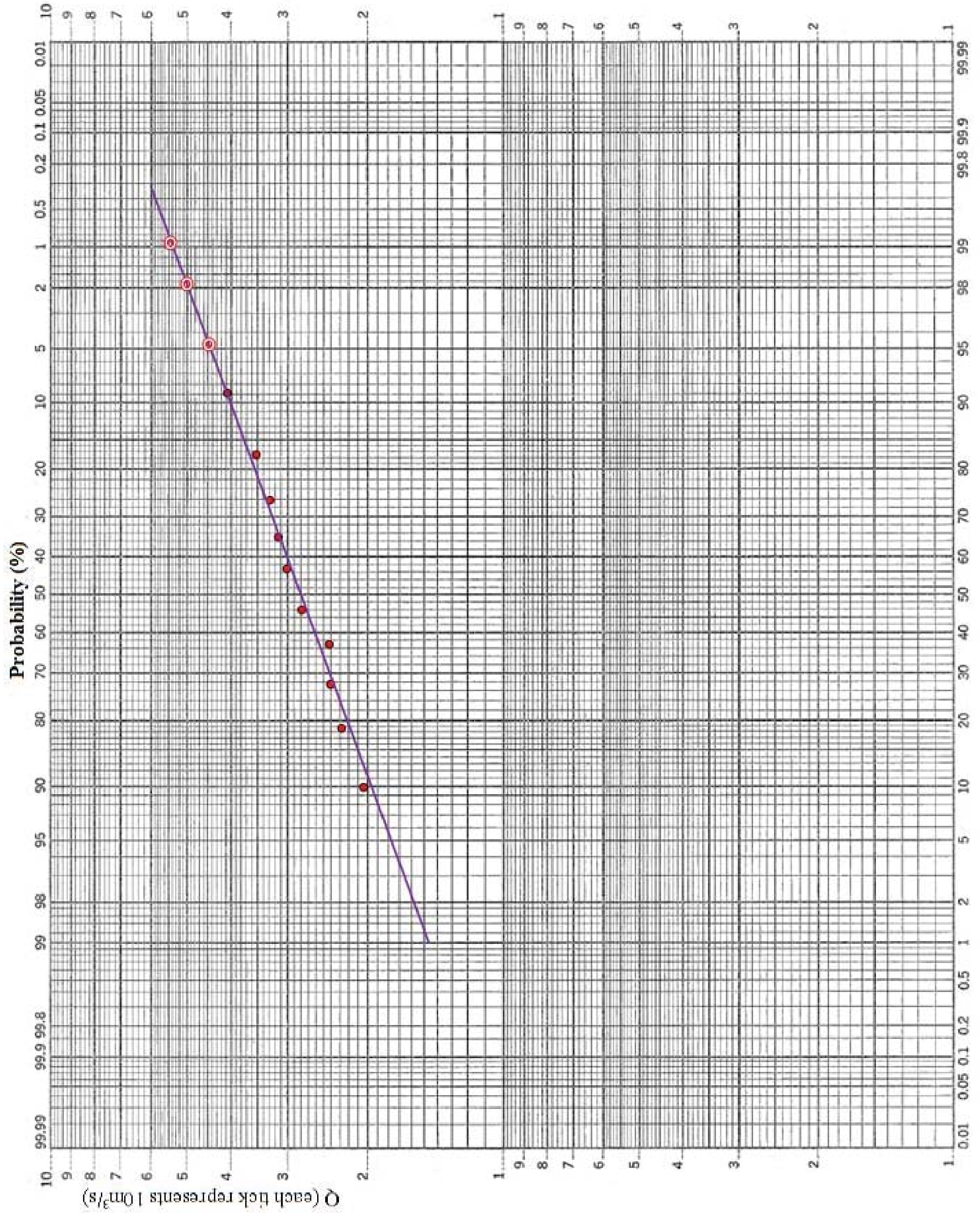
Therefore, the probability of such a flood occurring at least once in 40 years is
$1 - f(0;40,0.1) = 1 - 0.0148 = 0.9852$

2) Rank the data from largest to smallest and work out the plotting positions based on the Weibull equation.

Year	Q	Rank m	P=m/n+1 (%)
1987	25.1	8	73
1988	41.5	1	9
1989	29.9	5	45
1990	21.2	10	91
1991	35.5	2	18
1992	23.8	9	82
1993	25.5	7	64
1994	28	6	55
1995	33	3	27
1996	31.5	4	36

From the graph

T=20, 50, 100 years
P= 0.05, 0.02, 0.01
Q (m^3/s)= 45, 50, 53



Log 2 cycle probability paper

10. Hydrological Design

(reservoir and dam)

Hydrological design is used to choose key variables of water engineering systems, such as reservoir size, bridge span, dimension of spillway, etc. All projects are designed for the future and engineers are usually uncertain as to the precise conditions to which the works will be subjected. This is because that the exact sequence of stream flow for future years cannot be predicted and it is usually assumed that the future hydrological processes will follow the same patterns as their past ones. In this section, reservoir and dam design for water supply is used to illustrate the issues involved in hydrological design of water systems.

10.1 Reservoir and dam

A reservoir is an artificial lake to store water. Reservoirs are often created by dams which are made of concrete, earth, rock, or a mixture across a river. Once the dam is completed, the river fills the reservoir.

There are several types of reservoirs and three of them for water supply are: *a)* Direct supply reservoir: characterised by the impounding of a gravity inflow and the piping of outflow to supply; *b)* Pumped reservoir: reservoir inflow is from pumping. A pumped storage reservoir may be formed by damming a side valley to the main stream or by raising embankments to enclose a flat area in a river valley; *c)* Regulating reservoir: primarily impounding water for later release to a river when flows at some downstream abstract point would otherwise become too low. If the demand centre is downstream, there can be a large saving in aqueduct costs. The Bhatsai dam project for water supply to Bombay is such an example. In this section, we will further explore direct supply reservoir or regulating reservoir.

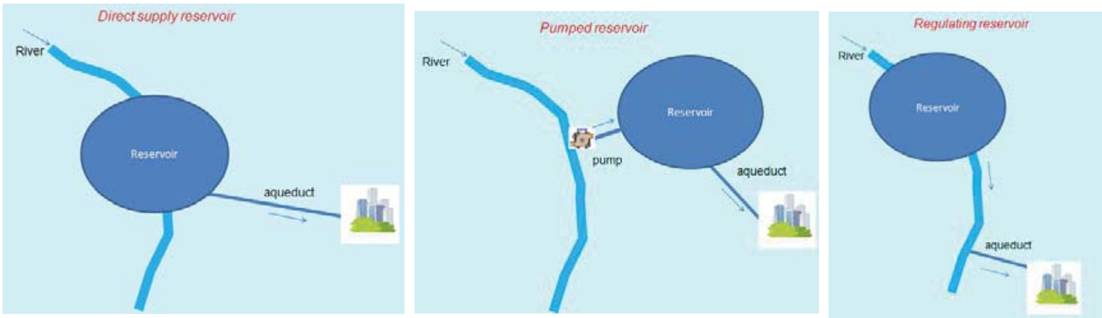


Figure 1 Reservoir types for water supply
a) direct supply reservoir, b) pumped reservoir and c) regulating reservoir

10.2 Basic design procedures

The procedures required to derive the reservoir storage and dam height for a water supply project can be carried out in the following steps. First, it is important to estimate the water demand based on the population and other factors. Second, a few potential dam sites are selected based on a contour map. It is important to check if enough river flow is available at the chosen sites to meet the demand. Depending on the catchment areas covering individual dam sites, different reservoir sizes are required and hence the corresponding dam heights. Three or four potential dam sites should be initially selected for hydrological analysis, and a final site will be decided on other factors (geological, economic and environment assessment, etc).

10.2.1 Water demand

Water demand is divided into
Domestic (In-house use, out-of-house use)
Trade (Industrial, commercial, institutional, ...)
Agricultural
Public (public park, fire fighting, ...)
Losses

Range of total consumption figures (litres per capita per day)

1. Highly industrialised cities (San Francisco, Philadelphia, ..)	600 - 700
2. Major cities (Glasgow, London,..)	400 - 500
3. Mixed cities with moderate industries (Liverpool, Plymouth, ..)	200-350
4. Mixed urban and rural areas with low proportion of industry (Brussels, ..)	150- 200
5. Small towns with little industrial demand	90-150

To calculate the water demand

Water demand = Safety factor × (Abstraction rate + Compensation flow)

where

Abstraction rate (water abstracted from the river) = population× water consumption

Population = the design population of the city to be supplied with water

Water consumption = water usage litres per capita per day

Compensation flow = minimum flow to be released from the reservoir. Compensation water is the flow that must be discharged below a direct supply reservoir to compensate the downstream water demand (people and ecosystems).

Safety factor =1.1~1.2

10.2.2 Catchment yield

To evaluate the hydrological feasibility of a potential dam site, a comparison between the water demand and catchment yield is necessary to check if sufficient water is available at the chosen site. A yield is the portion of the precipitation on a catchment that can be collected for use.

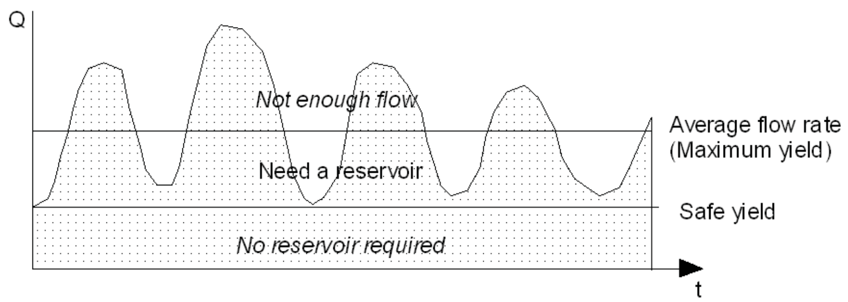


Figure 2 River hydrograph and Yield

Safe yield is the minimum yield recorded for a given past period. Abstraction is the intended or actual quantity of water withdrawn for use. Unless the minimum flow of stream is well above the minimum abstraction which must be satisfied in a water supply project, the minimum flow must be supplemented by water impounded in a reservoir. The firm yield is the mean annual rate of release of water through the reservoir that can be guaranteed. Naturally, the larger the reservoir storage, the greater is the firm yield, with the limit that the firm yield can never be greater than the mean inflow to the reservoir.

Since the firm yield can never be determined with certainty, it is better to treat yield in probabilistic terms. If the flow were absolutely constant, no reservoir would be required; but, as variability of the flow increases, the required reservoir capacity increases. This is another way of saying that a reservoir does not make water but merely permits its redistribution with respect to time.

10.2.3 Reservoir storage estimation

A reservoir is used to retain excess water from periods of high flow for use during periods of low flow. The impounding reservoirs have two functions: a) to impound water for beneficial use and b) to attenuate flood flows. An impounding reservoir presents a water surface for evaporation, and this loss should be considered for yield estimation. In addition, the possibility of large seepage losses should also be considered. People and ecosystem downstream may be entitled to have a certain amount of water that they may make their accustomed use of (compensation flow). Therefore, the water passed must be added to the abstraction or subtracted from the stream flow in calculating reservoir storage capacities.

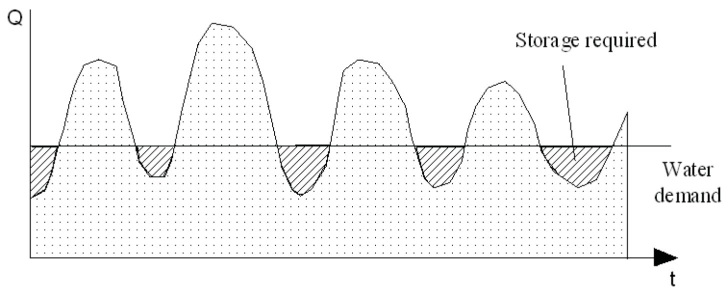


Figure 3 Reservoir storage required

There are three approaches that could be used to estimate the required reservoir storage.

a) Mass curve method (Rippl Diagram)
This method was developed by an Australian engineer in the 1890's to provide an answer to the question "… how big a reservoir is required for a given demand given an historic inflow sequence?"

A mass curve of supply is a curve showing the total (cumulative) volume entering a reservoir site over a certain time period (usually years). Records are examined for critical dry periods and the mass curve may be constructed for multiple years. Flow data at monthly increments are usually sufficient.

i) tabulate and plot accumulated flow, $\sum Q$ with time.
ii) compute the mean flow $\bar{Q}$
iii) add a demand line.
mass curve gradient > demand line gradient - reservoir filling
mass curve gradient > demand line gradient - reservoir emptying
iv) construct tangent to $\sum Q$ curve parallel to the demand line at all peaks and troughs (P and T). Ignore local maxima and minima. If the reservoir is full at P_1, it would need a capacity C1 to survive the period of emptying.
v) find the maximum *C*.
vi) from intersection points such as F_1, the reservoir will be spilling water over the spillway (assuming it was full at the previous P) until the next P point is reached.

Volume spilled = S (vertical heights)

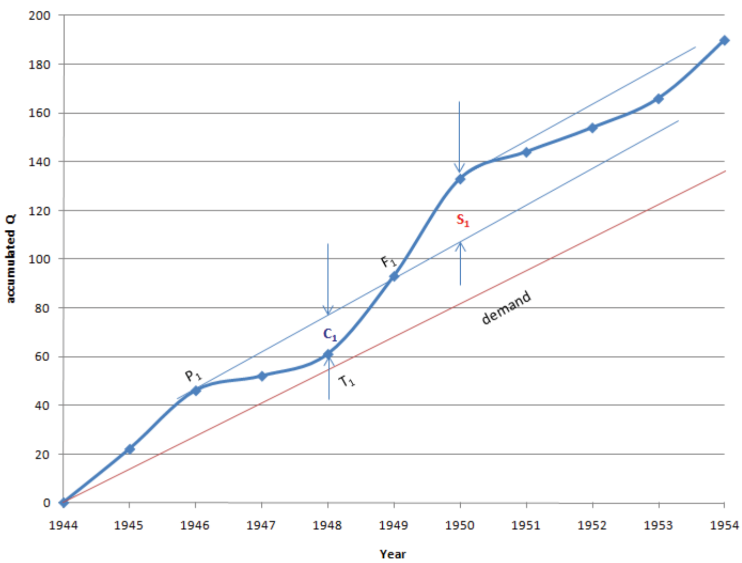


Figure 4 Mass curve (Rippl Diagram)

b) Water balance method
This method is similar to the Mass Curve method that it is also based on the past flow records. Instead of using a graph to derive reservoir storage information, water balance is applied with a table to solve the reservoir storage and spillage problem. The following table is for illustration only and there are many alternative ways to construct a water balance table.

No	Year	Q (m3/s)	demand	diff	accumulated	spillage	water in reservoir
0	1944	0	0	0	0		0
1	1945	22	13.6	8.4	8.4		8.4
2	1946	24	13.6	10.4	18.8	3.3	15.5
3	1947	6	13.6	-7.6	7.9		7.9
4	1948	9	13.6	-4.6	3.3		3.3
5	1949	32	13.6	18.4	21.7	6.2	15.5
6	1950	40	13.6	26.4	41.9	26.4	15.5
7	1951	11	13.6	-2.6	12.9		12.9
8	1952	10	13.6	-3.6	9.3		9.3
9	1953	12	13.6	-1.6	7.7		7.7
10	1954	24	13.6	10.4	18.1	2.6	15.5

c) Synthetic minimum flow method
This method is based on probability analysis and synthetic flow data instead of real flow data are used in the storage estimation. The procedure is formed as follows: 1) locate a long monthly flow record; 2) select the lowest monthly flows in each year; 3) rank the minimum monthly values starting with the driest; 4) convert flow in m3/s to m3 (i.e., flow rate into runoff volume); 5) calculate the return period by $T = (n+1)/m$ (In n year record, a record has been equal to or exceeded for m times); 6) the return periods should be plotted on a logarithmic paper; 7) draw a line that can best fit the data points; 8) read the value of 100 year return period from the fitted line (or any other specified return periods);

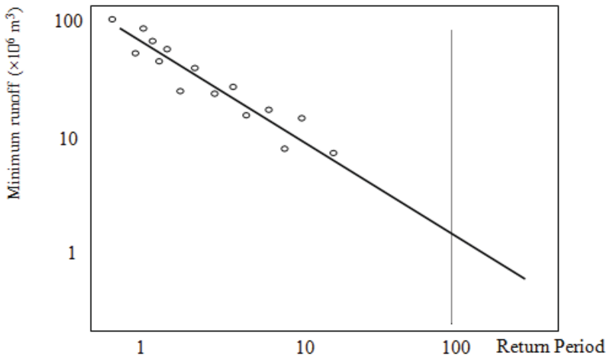


Figure 5 Monthly drought return period (with logarithmic plots, i.e., log-log)

The reservoir has to satisfy the water demand in a dry month with 100 year return period. In addition, the following month(s) might be very dry as well and the designers have to consider longer periods than just one dry month, so 2, or 3 or more months should be considered in the design (up to 11 months in this project). By repeating the procedures described above, it is possible to obtain a diagram as shown in Figure 6 and the flow values in 1, 2 , 3, 4, , …, 10, 11 months.

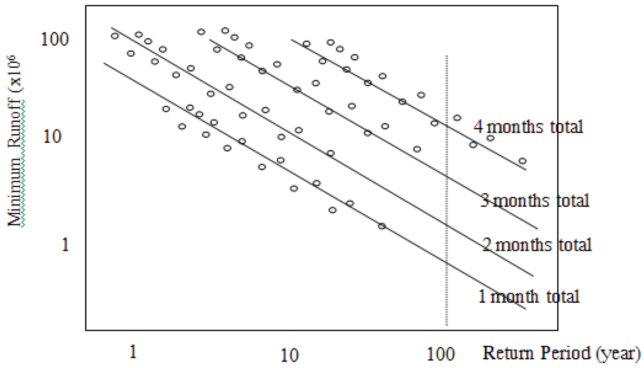


Figure 6. 1, 2,3, …, 11 month droughts (the logarithmic plot)

A synthetic mass curve can then be constructed from the cumulative minimum runoff data as shown in Figure 7. Each point is read from the logarithmic plots. If water demand is known, strike a tangent line (with the slope of the water demand) to the curve and the required storage can be found on the negative ordinate.

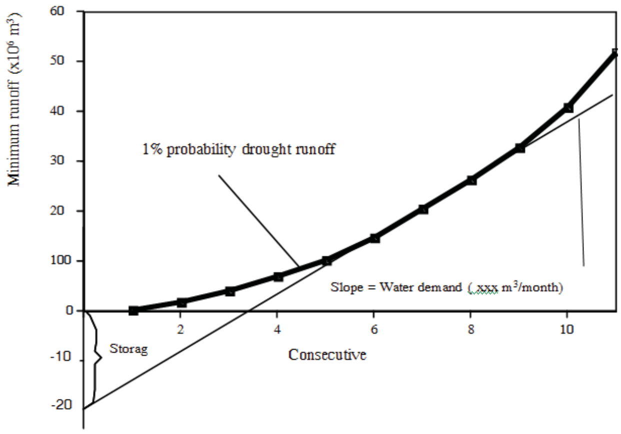


Figure 7 Minimum Runoff Diagram (for each dam site)

10.2.4 Dam height

Since the primary function of a reservoir is to provide water storage, its most important physical characteristic is storage capacity which is linked to the dam height. The relationship between a dam height and its reservoir storage capacity is usually described by a curve (Elevation – Storage curve, as shown in Figure 8) based on topographic surveys.

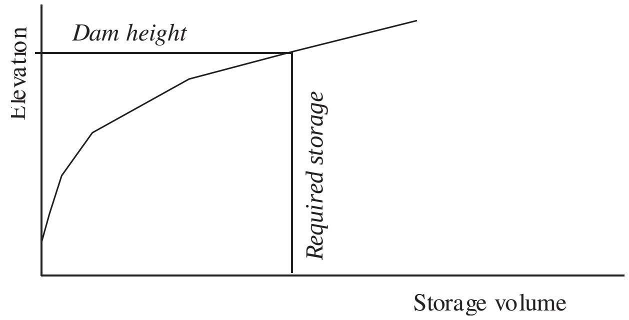


Figure 8 Elevation- Storage curve

The dam elevation for the required storage can be estimated from the Elevation- Storage curve. This elevation is referred to as the normal level that is the maximum elevation to which reservoir surface will rise during ordinary operating conditions (See Figure 9). For most reservoirs, their normal levels are determined by the elevations of spillway crests or tops of spillway gates. The minimum level is the lowest elevation to which a reservoir is to be drawn under normal conditions. This level may be fixed by the elevation of the lowest outlet in the dam. The storage volume between the minimum and normal level is called the useful storage. Water held below the minimum level is called the dead storage.

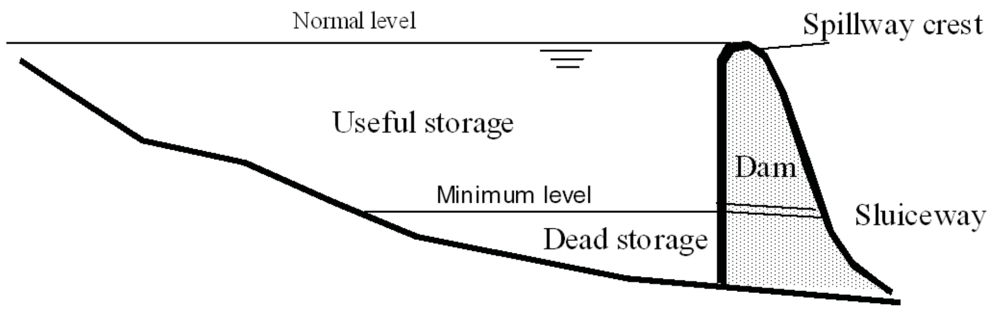


Figure 9 Zones of storage in a reservoir

Reservoirs described above are referred to as direct supply reservoirs or "conventional" reservoirs, and majority of reservoirs for water supply are in this category. The reservoir is filled by natural inflow from its catchment and water is drawn off through an aqueduct.

Questions 10

Hydrological Design

1. Describe the different types of reservoirs.

2. A water supply reservoir with a useful storage capacity of 4.9x108m^3 was formed by constructing a concrete dam across a river valley. It was designed to meet a demand of 13.6 cumecs and was completed and empty at the end of 1944. The average annual inflows for the ensuing 20 year period were 22, 24, 6, 9, 32, 40, 11, 10, 12, 24, 28, 6, 7, 9, 21, 16, 24, 27, 19 and 34 cumecs respectively. Use the water balance method to solve the following questions.
   a) When was the reservoir full for the first time?
   b) Estimate the number of months that the reservoir spilled over this 20 year period?
   c) Did the reservoir run dry during this period? If so, when and for how long?
   d) If the inflow data had been available prior to the original reservoir design what capacity would you have recommended?
   e) Given the existing storage determine the largest demand that could be sustained over the available historical record?

   (Answers: a) September 1946; b) 76.3 months; c) April 1958, 8.6 months; d) $5.9\times10^8 m^3$ *; e)* $12.5m^3/s$*)*

3. How to derive dam height from reservoir storage?

4. Sketch zones of storage in a reservoir.

Solutions 10

Hydrological Design

2. Total seconds in a year $T_{year} = 3600 \times 24 \times 365.25 = 3.16 \times 10^7$ seconds

Reservoir capacity is $\dfrac{4.9 \times 10^8}{3.16 \times 10^7} = 15.5 m^3 / s$ $\dfrac{4.9 \times 10^8}{3.16 \times 10^7} = 15.5 m^3 / s \cdot T_{year}$

15.5 is the limit

No	Year	Q (m^3/s)	demand	diff	accumulated	spillage	water in reservoir
0	1944	0	0	0	0		0
1	1945	22	13.6	8.4	8.4		8.4
2	1946	24	13.6	10.4	18.8	3.3	15.5
3	1947	6	13.6	-7.6	7.9		7.9
4	1948	9	13.6	-4.6	3.3		3.3
5	1949	32	13.6	18.4	21.7	6.2	15.5
6	1950	40	13.6	26.4	41.9	26.4	15.5
7	1951	11	13.6	-2.6	12.9		12.9
8	1952	10	13.6	-3.6	9.3		9.3
9	1953	12	13.6	-1.6	7.7		7.7
10	1954	24	13.6	10.4	18.1	2.6	15.5
11	1955	28	13.6	14.4	29.9	14.4	15.5
12	1956	6	13.6	-7.6	7.9		7.9
13	1957	7	13.6	-6.6	1.3		1.3
14	1958	9	13.6	-4.6	-3.3		0
15	1959	21	13.6	7.4	7.4		7.4
16	1960	16	13.6	2.4	9.8		9.8
17	1961	24	13.6	10.4	20.2	4.7	15.5
18	1962	27	13.6	13.4	28.9	13.4	15.5
19	1963	19	13.6	5.4	20.9	5.4	15.5
20	1964	34	13.6	20.4	35.9	20.4	15.5

a) The reservoir at the end of 1945 is 8.4 with a shortfall of 15.5-8.4= 7.1. The net inflow is 10.4, so each month has net inflow of 10.4/12=0.87. The months needed to fill up 7.1 deficit is 7.1/0.87=8.2 month, i.e., early September 1946

b) Number of months = 12 x spillage/inflow, therefore
(3.3/10.4+6.2/18.4+1+2.6/10.4+1+4.7/10.4+1+1+1)x12= 76.3 months

c) Yes, 1958, 1.3/4.6*12=3.4 months, hence April, lasted for 12x3.3/4.6= 8.6 months

d) Add 3.3 amount, hence 15.5+3.3=18.8m^3/s Tyear = $18.8\times 3.16\times 10^7 = 5.9\times 10^8 m^3$

e) The reservoir is full (15.5 m^3/s Tyear) at the end 1955, but Year 1956, 1957 and 1958 are very low (6+7+9)= 22 m^3/s, hence the maximum water supply is $(15.5+22)/3 = 12.5 m^3/s$

Appendix: Further Reading Resources

The following resources are highly recommended if you want to further explore the interested topics in hydrology. This is not an exhaustive list and will be updated regularly in the future. You are welcome to recommend useful books/web sites that are not on the list.

Chow, V.T., Maidment, D.R. and Mays, L.W. 1988, Applied Hydrology, McGraw-Hill
Connected Water Resources Project, 2009, http://www.connectedwater.gov.au/framework/hydrometric_k.php
CEH, 1999, Flood Estimation handbook, Centre for ecology and hydrology
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http://www.nerc-wallingford.ac.uk/ih/nrfa/spatialinfo/Index/introCatchmentSpatialInfo.html
Skeat, W.O. and Danerfield, B.J. 1969, Manual of British Water Engineering Practice, Fourth Edition, Volume II, Edited book, W. Heffer & Sons Ltd., Cambridge, England
Steel, E.W. and McGhee, T.J. 1979, Water Supply and Sewerage, Fifth Edition, McGraw-Hill Book Company, 1979
Twort, A.C., Law F.M., Crowley, F.W. and Ratnayaka, D.D. 1994, Water Supply, Edward Arnold, London, Fourth Edition
Viessman, W. and Lewis, G 1996, Introduction to Hydrology, HarperCollins College Publishers
Wikipedia, 2009, ‘hydrological cycle’, ‘Drainage basin’, ‘precipitation’, ‘Evaporation’, ‘Evapotranspiration’,’ Eddy covariance’, ‘groundwater’, ‘aquifer’, ‘water table’, ‘MODFLOW’, ‘FEMFLOW’, ‘MIKE SHE’, ‘hydrograph’, ‘time series, ‘Nyquist frequency’, ‘Aliasing’, 'Probability', ‘Return period’, reservoir’, etc.
Wilson, E.M. 1983, Engineering Hydrology, 3rd Edition, Macmillan Publishers, UK