Modelling Batch Systems Using Population Balances

Peter Dybdahl Hede

Contents

Modelling batch systems using population balances – A thorough introduction and review.

The present text introduces the use of population balances in the context of batch wet granulation and coating systems and it reviews the latest achievements and proposals in the scientific literature in this field. The nature and framework of population balance theory are highlighted and one-dimensional population balances are introduced from scratch. This may seems tedious at first but the derivation of even the simplest population balances is nevertheless not an easy task, and it is in fact difficult to find complete derivations elsewhere in literature. In addition to the introduction of the different terms in the population balances, a brief historical review of coalescence kernels for the one-dimensional balances is presented. Next, a brief introduction to multi-dimensional balances is given hereby presenting some of the recent suggestions of two- and four-dimensional balances. Further, population balance solution techniques are introduced in terms of two simple analytical solutions besides an introduction to the field of numerical solution techniques. A number of the latest advances in the various available numerical solution techniques are further presented.

The text is aimed at undergraduate university or engineering-school students working in the field of mathematical or chemical and biochemical engineering. Newly graduated as well as experienced engineers may also find relevant new information as emphasis is put on the newest scientific discoveries and proposals presented in recent years of scientific publications. It is the hope that the present introductory text will be helpful to the reader – particularly in the first stages of the process of working with population balances. The comprehensive literature list may also hopefully be an inspiration for further reading.

I alone am responsible for any misprints or errors and I will be grateful to receive any critics and/or suggestions for further improvements.

Copenhagen, September 2006

Peter Dybdahl Hede

1 Modelling Batch Fluid Bed Granulation Processes Using Population Balances

In many types of chemical processes, particles are vital parts of the system whether or not the particles are present naturally or engineered into the system. Such particle containing systems are often referred to as dispersed phase system (Ramkrishna, 2000). A granulating batch fluid bed system is a typical dispersed phase system in that the properties of the particles play a significant role in the behaviour of the entire system. This is due to the continually formation as well as breakage of agglomerates and the resulting influence on fluidisation and heat transfer characteristics. As two particles will not be exactly of the same size, the particle system must be characterised by a distribution of sizes and the particle size distribution is often considered as a parameter to characterise such systems in a simple uniform manner (Randolph & Larson, 1988).

Any analysis of granulating systems seeks to synthesise the behaviour of the population of particles and its environment from the single particle behaviour in their local environment. The population is described by the density of a suitable extensive variable being usually the number of particles but sometimes by other variables such as mass or volume of the particles. A population balance equation is thereby an equation in the number density and may be regarded as representing a number balance on particles of a particular state. (Ramkrishna, 2000 and Iveson, 2002). In other words, the population balance keeps track of the particle size distribution at any time during the process and in addition (if being fully developed) describes all the changes in the particle size distribution due to the various possible mechanisms that particles can undergo during granulation. A description or modelling of granulating systems by means of population balances is thereby a description in terms of a model of how the particle size distribution changes with granulation time.

1.1 The nature and framework of population balances

The particles in population balances are characterised by internal as well as external coordinates. The internal coordinates of the particle provide quantitative characteristics of the particle properties such as size, shape and porosity etc. (Cameron et al., 2005). Commonly, internal coordinates are defined in terms of a vector $\mathbf{x} \equiv (x_1, x_2, \ldots, x_z)$ in which z represents z different physical quantities associated with the particle. Each physical quantity is thereby formally given its own dimension in the **x** vector. The external coordinates denote and specify the location of the particles in physical space. Hence the external coordinates denote the position vector inside the fluid bed in terms of Euclidian coordinates being rectangular, cylindrical or spherical coordinates. Traditionally, external coordinates are defined in terms of a vector $\mathbf{r} \equiv (r_1, r_2, r_3)$. One or more of either the internal or external coordinates may in principle be discrete while the others may be continuous. In the case of batch fluid bed systems, all the external coordinates are continuous[i].

The joint space of internal and external coordinates is commonly referred to as the *particle state space*. Fundamental to the formulation of population balances is the assumption that there exists a number density of particles at every point in the particle state space and that the number of particles in any region of the state space is obtained by integrating the number density over the region desired. In a discrete region, the integration amounts to simply summing over the discrete state in the specific region. The population balance equation is an equation in the number density and may be regarded as representing a number balance on particles of a particular state.

The population balance equation can be coupled with conservation equations for entities in the particle environmental or continuous phase, as the usual well-known transport equations expressing conservation laws for material systems apply to the behaviour of single particles and their binary interactions (Ramkrishna, 2000). In the sense of batch fluid bed granulation, this could e.g. be the kinetic expressions for each mechanism responsible for changing the particle size (Cryer, 1999). Population balances are thereby in other words models describing how the number of individual particles in a population and their properties change with time and the conditions of growth and breakage (Hjortsø, 2006). The population balance equation basically accounts for the various ways in which particles of a specific state can either form or disappear from the system. A population balance follows the change in the particle size distribution as granules are formed, broken, grow and enter or leave the control volume (Cryer, 1999). In words, the full population balance for a batch fluid bed may be expressed as (Sheffield, 2005):

The rate of change of numbers of particles in a size range =

The rate at which particles grow into that size range by coating (layering)

+ The rate at which particles are "born" in that size range by aggregation of smaller particles

– The rate at which particles "die" in that size range by aggregating into large size ranges (1.1)

+ The rate at which particles are "born" from breakage of larger size ranges

– The rate at which particles in that size range "die" from breakage into smaller size ranges.

Inside the fluid bed, the particles may interact between themselves as well as with the continuous phase during processing. Such behaviour may vary from particle to particle depending on a number of properties that may be associated with the particle. The variables representing such properties may be either discrete or continuous. There are indeed many practical reasons for representing continuous variables in terms of discrete levels. E.g. is it often convenient to represent a continuous particle size distribution in terms of discrete mesh sizes. Later sections concerned with solutions of population balances will indeed make use of these principles but towards the derivation of the general population balance, the primary concern is on continuous variables, which nevertheless often are encountered in population balance

analysis. One example of a continuous variable is the temperature of a particle inside the fluid bed. Likewise is the granulation time regarded as a continuous variable.

A fundamental assumption in the following analysis is that the rate of change of state of any particle is a function only of the state of the particle in question and the local continuous phase. This implies that the possibility of direct interaction between two particles is excluded meaning that two particles interact only via the continuous phase. In order to enable such a local characterisation of the continuous phase variables, it is necessary to assume that the particles are much smaller than the length scale in which the continuous phase quantities vary (Ramkrishna, 2000). This is usually no problem in fluid bed granulation where the particle diameters are much smaller than the diameter of the fluidisation chamber in most practical situations.

The particle state is commonly represented by a finite particle state vector[ii] referred to as (**x**, **r**), which accounts for both the internal and external coordinates. The components of the particle state vector is chosen depending on the variables needed to specify the situation of interest. That means that the dimensionality of the particle state vector is chosen by the modeller's interest. It is common to let the domain of the internal coordinates be represented by $\Omega_{\mathbf{x}}$ and the domain of external coordinates be represented by $\Omega_{\mathbf{r}}$, which thereby is the set of points in the particle state space in which the particles are present. These domains may be bounded or have infinite boundaries depending on the choice of components in the particle state vector. The particle population may be regarded as randomly distributed in the particle state space, which includes physical space as well as the space of internal coordinates.

The continuous phase variables, which affect the behaviour of each particle, are commonly collated into a finite c-dimensional vector field and hence the continuous phase vector is defined as (Ramkrishna, 2000):

$$\mathbf{Y}(\mathbf{r},t) \equiv \left[Y_1(\mathbf{r},t), Y_2(\mathbf{r},t), Y_3(\mathbf{r},t), ..., Y_c(\mathbf{r},t)\right], \mathbf{r} \in \Omega_{\mathbf{r}} \tag{1.2}$$

which thereby obviously is a function of the external coordinates **r** and time only. The evolution of this c-dimensional vector field is governed by the laws of transport phenomena as well as the interaction between particles. In some population balance problems, it is not necessary to account for any continuous phase balance and thereby it is not necessary to include the continuous phase variables, as the interaction between the population and the continuous phase may not bring about any change in the continuous phase. This is however not the case in batch fluid bed granulation where the change in e.g. particle size distribution will significantly influence the fluidisation flow pattern in the continuous phase. All of the phenomena and equations that actually govern the continuous phase vector will one way or the other involve the number density of particles in the particulate phase. The average number density function is defined on the particle state space as:

$$E\left[n(\mathbf{x},\mathbf{r},t)\right] \equiv f_1(\mathbf{x},\mathbf{r},t)\,,\ \mathbf{x} \in \Omega_{\mathbf{x}}\,,\ \mathbf{r} \in \Omega_{\mathbf{r}} \tag{1.3}$$

in which the left-hand side of equation 1.3 denotes the average of the actual number density $n(\mathbf{x},\mathbf{r},t)$, while the right-hand side displays the average number density function $f_1(\mathbf{x},\mathbf{r},t)$.

The definition in equation 1.3 implies that the average number of particles in the infinitesimal volume $dV_{\mathbf{x}}dV_{\mathbf{r}}$ in the particle state space about the particle state $(\mathbf{x},\mathbf{r})$ is $f_1(\mathbf{x},\mathbf{r},t)dV_{\mathbf{x}}dV_{\mathbf{r}}$. However to save some notation is has become common to loosely refer to particles in the volume of $dV_{\mathbf{x}}dV_{\mathbf{r}}$ about the particle state $(\mathbf{x},\mathbf{r})$ merely as *particles of state (**x**,**r**)*, although this is not entirely correct. The average number density function $f_1(\mathbf{x},\mathbf{r},t)$ is assumed to be sufficiently smooth to allow differentiation with respect to any of its arguments as many times as it may become necessary. The average number density in equation 1.3 allows the calculation of the average number of particles in any region of the particle state space according to:

$$\int_{\Omega_x} dV_{\mathbf{x}} \int_{\Omega_r} dV_{\mathbf{r}} f_1(\mathbf{x},\mathbf{r},t) \tag{1.4}$$

Analogously, the average total number of particles per unit volume of physical space, N(**r**,t), may be found as:

$$N(\mathbf{r},t) = \int_{\Omega_r} dV_{\mathbf{r}} f_1(\mathbf{x},\mathbf{r},t) \tag{1.5}$$

Having introduced the framework and basic parameters of the population balance equation it now becomes possible to set up the balances, which is the subject for the coming sections.

1.2 One-dimensional population balances for fluid bed batch systems[iii]

Although the various definitions of vectors and functions in the previous section allows the derivation of a general multi-dimensional population balance, most work with population balances for fluid bed systems only considers one-dimensional balances in which only one internal parameter varies. In the following sections such one-dimensional balances will be derived.

1.2.1 One-dimensional growth and aggregation balances

In the derivation of the one-dimensional case, one considers a population of particles distributed according to their size x, which is often taken to be the mass (or volume) of the particle. The parameter x is allowed to vary between 0 and ∞. It is further assumed that the particles are uniformly distributed in space so that the average number density function $f_1(\mathbf{x},\mathbf{r},t)$ is independent of external coordinates, and as there is only one internal coordinate, the average number density function in the one-dimensional case reduces to[iv] $f_1(x,t)$. The growth rate of particles due to coating (layering) of size x is represented by G(x,t) and for the time being, it is assumed that particles will only change in size due to growth by coating. The particles may then be viewed as distributed along the size coordinate x and embedded on a string deforming with velocity G(x,t). Next, an arbitrary size region [a,b] on the stationary size coordinate x is chosen with respect to which the string with the imbedded particles is deforming. What is interesting in this size interval is the rate of change of the number of particles. As the string deforms, particles commute through the interval [a,b] across the interval end-points a and b, thereby changing the number of particles in the interval. The rate of change in the number of particles in [a,b] caused by this transport at a and b is given by:

$$G(a,t) \cdot f_1(a,t) - G(b,t) \cdot f_1(b,t) \tag{1.6}$$

in which the first term represents the particle flux in at a and the second term represents the particle flux out at b. It may be assumed for the present that this is the only way in which the

number of particles in the interval [a,b] can change. That is birth and death of particles is no yet considered[v]. With the above mentioned assumptions the number balance in the interval [a,b] may be stated according to:

$$\frac{d}{dt}\int_a^b f_1(x,t)dx = G(a,t)\cdot f_1(a,t) - G(b,t)\cdot f_1(b,t) \qquad (1.7)$$

Which may be rewritten as:

$$\int_a^b \left[\frac{\partial f_1(x,t)}{\partial t} + \frac{\partial}{\partial x}(G(x,t) \cdot f_1(x,t)) \right] dx = 0 \tag{1.8}$$

As mentioned earlier, the average number density $f_1(\mathbf{x},\mathbf{r},t)$ is generally assumed to be sufficiently smooth to allow differentiation with respect to any of its arguments as many times as it may become necessary. That is assumed to hold for G(x,t) as well and as the interval [a,b] is chosen arbitrarily, the smoothness of the integrand implies that the integral vanishes altogether. Hence the one-dimensional population balance may be found as:

$$\underbrace{\frac{\partial f_1(x,t)}{\partial t}}_{\text{Average number density}} + \underbrace{\frac{\partial}{\partial x}(G(x,t) \cdot f_1(x,t))}_{\text{Growth due to coating (layering)}} = 0 \tag{1.9}$$

This derived population balance must be supplemented with initial as well as boundary conditions. If it is assumed that initially no particles of size x existed, the initial condition is obviously:

$$\text{IC: } f_1(x,0) = 0 \tag{1.10}$$

For the boundary condition we let the nucleation rate be n_0 particles per unit time and assume that the newly formed particles have size and mass zero. This rate should be the same as the particle flux in at the beginning of the string at x = 0. This means that the required boundary condition can be stated as:

$$\text{BC: } G(0,t) f_1(0,t) = \dot{n}_0 \tag{1.11}$$

If equation 1.9 is integrated over all of the different particle masses, one obtains the following equation:

$$\text{Total Balance: } \frac{d}{dt} \int_0^\infty f_1(x,t) \cdot dx = G(0,t) \cdot f_1(0,t) - G(\infty,t) \cdot f_1(\infty,t) = \dot{n}_0 \tag{1.12}$$

The equality on the extreme right results from the fact that particles can increase in number in this simplified case only by nucleation. Comparing equation 1.11 with equation 1.12 indicates that:

$$\text{Regularity condition: } G(\infty,t) f_1(\infty,t) = 0 \tag{1.13}$$

which might seem strange at first. Equation 1.13 does however not insist that the number density itself vanishes at infinite mass if the growth rate vanishes for large particles, but rather that the number density must vanish for arbitrary large sizes if the growth rate does not vanish for large particles. Equation 1.13 is sometimes referred to as the *regularity condition* (Ramkrishna, 2000).

In the derivation of the one-dimensional population balance, the influence of birth and death of particles as well as the influence of environment was neglected and the derived equation 1.9 may only be applied for pure growth processes. As stated in previous chapters, population balances not accounting for these birth and death terms due to agglomeration may only have a little chance of successfully describing the particle size distribution during fluid bed granulation. So far, the net rate of generation of particles due to agglomeration in the size range of x to x+dx will be described by h(x,t)dx, where the identity h(x,t) depends on the models of agglomeration as it will be presented later. With the introduction of the net rate of generation, it becomes obvious that equation 1.8 must be extended according to:

$$\int_a^b \left[\frac{\partial f_1(x,t)}{\partial t} + \frac{\partial}{\partial x}(G(x,t) \cdot f_1(x,t)) - h(x,t) \right] dx = 0 \tag{1.14}$$

The resulting population balance thereby by analogous treatment to equation 1.8 becomes:

$$\underbrace{\frac{\partial f_1(x,t)}{\partial t}}_{\text{Average number density}} + \underbrace{\frac{\partial}{\partial x}(G(x,t) \cdot f_1(x,t))}_{\text{Growth due to coating (layering)}} = \underbrace{h(x,t)}_{\text{Net rate of generation}} \tag{1.15}$$

As before, the preceding population balance equation must be supplemented with boundary conditions. Equation 1.11 may still serve as boundary condition whereas the regularity condition in equation 1.16 holds for the total balance analogously to equation 1.12 and equation 1.13:

$$\text{Total Balance: } \frac{d}{dt}\int_0^\infty f_1(x,t) \cdot dx = \dot{n}_0 + \int_0^\infty h(x,t)dx \tag{1.16}$$

The derived equation 1.15 will be the basis for further development of the one-dimensional population balance. So far the term h(x,t) has been introduced as the net rate of generation of particle with size x but the single h(x,t) term consists in fact of a birth term $h_a^+(x,t)$ as well as a death term $h_a^-(x,t)$. In the following, these two terms will be developed in terms of the previously introduced average number density function and a new term called the coalescence frequency or aggregation frequency[vi]. The aggregation frequency represents the probability per unit time that a pair of particles of specified states will aggregate. Alternatively, it could be seen as representing the fraction of particles pairs of specified states aggregating per unit time. This interpretation must however to some extent be modified as it is a well-established assumption

that a particle population in a fluid bed is considered well-mixed implying that external coordinates do not appear explicitly in the population density.

A further basic assumption in the following is that the continuous phase is sufficiently dilute to make only binary aggregation significant[vii]. It is obviously that this assumption will not be true for crowded systems in which it is conceivable that several adjacent particles could simultaneously aggregate. Most fluid beds are relatively loosely packed which implies that particles of type Geldart B and Geldart C in fact flow as single particles (Hoomans, 1999). Hence the binary aggregation assumption seems reasonable for fluid bed purposes although being indeed a rough assumption.

In the development of the birth and death term, it is assumed further that a particle is described by the particle state vector (**x**, **r**) in a continuous phase of state **Y** implying that it can be described by the continuous phase vector **Y**(**r**, t). The probability that a particle of state (**x**, **r**) and another particle of state (**x**′, **r**′) will aggregate in the time interval t to t+dt is defined as:

$$\beta_{dt}(\mathbf{x}, \mathbf{r}; \mathbf{x}', \mathbf{r}'; \mathbf{Y}, t) \cdot dt \tag{1.17}$$

The defined term may thereby be seen as the fraction of particle pairs of states (**x**, **r**) and (**x**′, **r**′) aggregating per unit time. The aggregation frequency is thereby defined for an ordered pair of particles although from a physical viewpoint, the ordering of particle pairs should not alter the value of the frequency. That means that equation 1.17 satisfies the symmetry property being:

$$\beta_{dt}(\mathbf{x}, \mathbf{r}; \mathbf{x}', \mathbf{r}'; \mathbf{Y}, t) \cdot dt = \beta_{dt}(\mathbf{x}', \mathbf{r}'; \mathbf{x}, \mathbf{r}; \mathbf{Y}, t) \cdot dt \quad (1.18)$$

It is nevertheless essential to consider only one of the above order for a given pair of particles. The explicit time dependence in equation 1.17 is not a desirable feature in models and is thereby eliminated in the remaining sections according to:

$$\beta_{dt}(\mathbf{x}, \mathbf{r}; \mathbf{x}', \mathbf{r}'; \mathbf{Y}, t) \cdot dt = \beta(\mathbf{x}', \mathbf{r}'; \mathbf{x}, \mathbf{r}; \mathbf{Y}, t) \quad (1.19)$$

The above definitions and assumptions are in fact generally valid but to follow the previous trend of the one-dimensional population balance, it is again assumed that the population of particles is distributed according to only one internal coordinate x (e.g. mass) and that environmental effects can be neglected. Likewise is the contribution to birth from particle nucleation only accounted for in the boundary conditions. This means that the aggregation frequency for particle pairs of mass x and x′ can be denoted β(x, x′), which again depends on the form of relative motion between the particles. Aggregation at t between particles of mass x and mass x′ is assumed to be proportional to the product of the number densities $f_1(x,t)$ and $f_1(x',t)$. In the birth of particles with mass x, particles of mass x-x′ aggregates with particles of mass x′ to produce particles of mass x. This follows from the fact that mass is conserved in a collision. Clearly, as x′ varies between 0 and x so does x-x′ meaning that each pair in the set is considered twice. Hence the total birth of particles with mass x may be expressed as:

$$\text{Birth of particle with mass x}: \quad h_a^{+}(x,t) = \frac{1}{2}\int_0^x \beta(x - x', x') \cdot f_1(x - x', t) \cdot f_1(x', t)dx' \quad (1.20)$$

in which the factor ½ ensures that for a given x′, the collision between particles of mass x′ and mass x-x′ is not counted twice. The integral from 0 to x ensures that all aggregating particles resulting in agglomerates of mass x will be included in the $h_a^{+}(x,t)$ term. Similar to the birth of particle of mass x, the death of particle of mass x due to agglomeration is given by:

$$\text{Death of particle with mass x}: \quad h_a^{-}(x,t) = f_1(x,t)\int_0^\infty \beta(x, x') \cdot f_1(x', t)dx' \quad (1.21)$$

in which the integral from 0 to infinite ensures that the possibility that particles of mass x aggregates with any particle other particle with mass ranging from 0 to infinity is accounted for.

With the introduction of the birth and death functions, the net rate of generation of particles of size x, h(x,t), may be written as:

$$h(x,t) = h_a^{+}(x,t) - h_a^{-}(x,t) \quad (1.22)$$

Equation 1.15 may hereby be extended resulting in the one-dimensional population balance according to:

$$\underbrace{\frac{\partial f_1(x,t)}{\partial t}}_{\text{Average number density}} + \underbrace{\frac{\partial}{\partial x}(G(x,t)\cdot f_1(x,t))}_{\text{Growth due to coating (layering)}} = \underbrace{\frac{1}{2}\int_0^x \beta(x-x',x')\cdot f_1(x-x',t)\cdot f_1(x',t)dx'}_{\text{Birth due to aggregation}} - \underbrace{f_1(x,t)\int_0^\infty \beta(x,x')\cdot f_1(x',t)dx'}_{\text{Death due to aggregation}} \quad (1.23)$$

The resulting equation 1.23 thus appears as an integro-differential equation in the number density function $f_1(x,t)$. The model does account for growth as well as birth and death of new particles of mass or size class x due to agglomeration although phenomena such as breakage or environmental effects are completely neglected.

Equation 1.23 is in principle the original population balance equation introduced by Hulburt & Katz (1964), which in some sense introduced population balances in the field of granulation. As it will become evident in later sections, a number of problems are associated with the use of equation 1.23 as it is to some extent a too simplified an approach for detailed granulation studies. Despite the obvious limitations of the balance, roughly more than 3/4 of the available literature in batch fluid bed granulation in fact uses the one-dimensional balance in equation 1.23. Some of the reasons for this will be presented in section 1.2.4, but in the following section, the one-dimensional population balance will be expanded to include breakage phenomena.

1.2.2 Modification of the one-dimensional PB – inclusion of breakage functions

Although the influence of breakage on the number density function has often been neglected in low agitative systems as fluid beds, a more precise version of equation 1.23 must include terms that account for birth and death rates due to breakage. Modifications to equation 1.23 have been presented by Ramkrishna (2000) and Cameron et al. (2005) in order to include the birth of new particles in size class x due to breakage of particles from higher size classes, h_b^+, and the death of particle of size class x due to breakage into lower size classes, h_b^-.

In the derivation of the breakage birth and death terms it is assumed that the break-up of particles occurs independently of each other. This assumption may seem reasonable at first, but quite a few articles regarding breakage of particles in fluid beds (e.g. Beekman, 2000) indicates clearly that the extent and type of particle breakage is highly affected by particle-particle interaction. Hence, any break-up equation should reasonably be highly dependent of how many other particles that are present nearby the particle in question and particularly on how large and with what speed these particles are impacting the particle. Including these unknowns will nevertheless expand the breakage terms resulting in equations impossible to solve. This independent particle breakage assumption will thereby be maintained in the following.

Additionally, it is assumed that breakage occurs instantaneous implying that it occurs on a small time scale compared with the time scale used for observing the changes in particle population.

Further, b(**x**, **r**, **Y**, t) is introduced as the specific breakage rate of particles of state (**x**, **r**) at time t in a continuous phase of state **Y**. It represents the fraction of particles of state (**x**, **r**) breaking per unit time and may thereby also be seen as a breakage frequency. The introduction of the breakage frequency thereby gives the average number of particles of state (**x**, **r**) lost by breakage per unit time according to:

$$h_b^-(\mathbf{x},\mathbf{r},\mathbf{Y},t) = b(\mathbf{x},\mathbf{r},\mathbf{Y},t)\cdot f_1(\mathbf{x},\mathbf{r},t) \qquad (1.24)$$

In the derivation of the birth term due to breakage, two quantities are commonly introduced: $\nu(\mathbf{x}', \mathbf{r}', \mathbf{Y}, t)$ describes the average number of particles formed from the break-up of a single particle of state ($\mathbf{x}'$, $\mathbf{r}'$) in an environment of state **Y** at time t, whereas $P(\mathbf{x}, \mathbf{r}| \mathbf{x}', \mathbf{r}', \mathbf{Y}, t)$ is the probability density function for particles that have state (**x**, **r**) resulting from the break-up of a particle of state ($\mathbf{x}'$, $\mathbf{r}'$) in an environment of state **Y** at time t. The probability density function is thereby a continuous distributed fraction over the particle state space.

The average number of particles $\nu(\mathbf{x}', \mathbf{r}', \mathbf{Y}, t)$ depends obviously on the mechanical properties of the particles and is frequently known from separate breakage studies. It obviously has a minimum value of two but it is not restricted to be an integer as it is an average number.

The probability density function P(**x**, **r**| **x**′, **r**′, **Y**, t) representing the distribution of particle states for the fragments resulting from breakage is also often determined quantitatively from experimental studies. The function inherits certain properties from conservation laws, which must constrain the breakage process. First of all the normalisation condition must be satisfied:

$$\int_{\Omega_x} P(\mathbf{x}, \mathbf{r} \mid \mathbf{x}', \mathbf{r}', \mathbf{Y}, t) dV_{\mathbf{x}} = 1 \tag{1.25}$$

If m(**x**) represents the mass of a particle of internal state **x,** then the conservation law of mass would require that:

$$P(\mathbf{x}, \mathbf{r} \mid \mathbf{x}', \mathbf{r}', \mathbf{Y}, t) = 0 \ , \ m(\mathbf{x}) \geq m(\mathbf{x}') \tag{1.26}$$

Additionally, the following relation must be satisfied:

$$m(\mathbf{x}') \geq \nu(\mathbf{x}', \mathbf{r}', \mathbf{Y}, t) \cdot \int_{\Omega_x} m(\mathbf{x}) \cdot P(\mathbf{x}, \mathbf{r} \mid \mathbf{x}', \mathbf{r}', \mathbf{Y}, t) dV_{\mathbf{x}} \tag{1.27}$$

in which the equality holds if there were no loss of mass during breakage. This last relation represents the obvious principle that the mass of all fragments within the system formed from breakage of a parent particle must be no more than the mass of the parent particle.

Having defined the average number of particles ν(**x**′, **r**′, **Y**, t) and the probability density function P(**x**, **r**| **x**′, **r**′, **Y**, t), the birth of particles of state (**x**, **r**) by breakage of particles of all particle states being internal and external may be expressed according to:

$$h_b^{+}(\mathbf{x}, \mathbf{r}, \mathbf{Y}, t) = \int_{\Omega_r} dV_{\mathbf{r}} \int_{\Omega_x} dV_{\mathbf{x}} \cdot \nu(\mathbf{x}', \mathbf{r}', \mathbf{Y}, t) \cdot b(\mathbf{x}', \mathbf{r}', \mathbf{Y}, t) \cdot P(\mathbf{x}, \mathbf{r} \mid \mathbf{x}', \mathbf{r}', \mathbf{Y}, t) \cdot f_1(\mathbf{x}', \mathbf{r}', t) \tag{1.28}$$

The integrand on the right-hand side of equation 1.28 is obtained as follows: The number of particles of state (**x**′, **r**′) that break up per unit time is $b(\mathbf{x}', \mathbf{r}', \mathbf{Y}, t) \cdot f_1(\mathbf{x}', \mathbf{r}', t) \cdot dV_{\mathbf{x}} \cdot dV_{\mathbf{r}}$ thereby producing new particles numbering $\nu(\mathbf{x}', \mathbf{r}', \mathbf{Y}, t) \cdot b(\mathbf{x}', \mathbf{r}', \mathbf{Y}, t) \cdot f_1(\mathbf{x}', \mathbf{r}', t) \cdot dV_{\mathbf{x}} \cdot dV_{\mathbf{r}}$ of which P(**x**, **r**| **x**′, **r**′, **Y**, t) represents the fraction of particles of state (**x**, **r**).

Following the trends from the previous section thereby neglecting any environmental effects as well as to assume that the population of particles is distributed according to only one internal coordinate x (e.g. mass) equation 1.24 and equation 1.28 obviously becomes:

$$h_b^{-}(x, t) = b(x, t) \cdot f_1(x, t) \tag{1.29}$$

$$h_b^{+}(x,t) = \int_x^{\infty} \upsilon(x') \cdot b(x') \cdot P(x \mid x') \cdot f_1(x',t)dx' \tag{1.30}$$

The original term describing the net rate of generation of particles of size x, h(x,t), in equation 3.22 may thereby be expanded in order to include any breakage phenomena in addition to agglomeration according to:

$$h(x,t) = h_a^{+}(x,t) - h_a^{-}(x,t) + h_b^{+}(x,t) - h_b^{-}(x,t) \tag{1.31}$$

The resulting one-dimensional population balance accounting for aggregation, growth as well as breakage may thereby by analogous manipulation to the derivation of equation 1.23 be expressed according to:

$$\underbrace{\frac{\partial f_1(x,t)}{\partial t}}_{\text{Average number density}} + \underbrace{\frac{\partial}{\partial x}(G(x,t) \cdot f_1(x,t))}_{\text{Growth due to coating (layering)}} =$$

$$\underbrace{\frac{1}{2}\int_0^x \beta(x-x',x') \cdot f_1(x-x',t) \cdot f_1(x',t)dx'}_{\text{Birth due to aggregation}} - \underbrace{f_1(x,t)\int_0^{\infty} \beta(x,x') \cdot f_1(x',t)dx'}_{\text{Death due to aggregation}} \tag{1.32}$$

$$+ \underbrace{\int_x^{\infty} \upsilon(x') \cdot b(x') \cdot P(x \mid x') \cdot f_1(x',t)dx'}_{\text{Birth due to breakage}} - \underbrace{b(x,t) \cdot f_1(x,t)}_{\text{Death due to brekage}}$$

Even though equation 1.32 is the most advanced of the one-dimensional balances, and thereby ought to give the most accurate and versatile description of the change in particle size distribution during fluid bed granulation, it is nevertheless rarely used, as an adequate description of the probability density function P(**x**, **r**| **x**′, **r**′, **Y**, t) is difficult to obtain with real granulating fluid beds.

1.2.3 Review of coalescence kernels for one-dimensional balances

In the earlier development of the birth and death terms the parameter β was introduced expressing the probability that a particle of state (**x**, **r**) and another particle of state (**x**′, **r**′) will aggregate in the time interval t to t+dt according to:

$$\beta_{dt}(\mathbf{x},\mathbf{r};\mathbf{x}',\mathbf{r}';\mathbf{Y},t) \cdot dt = \beta(\mathbf{x}',\mathbf{r}';\mathbf{x},\mathbf{r};\mathbf{Y},t) \tag{1.33}$$

So far the aggregation frequency has been expressed in terms of the single parameter β but this parameter obviously depends on a number of physical and statistical properties associated with the collision. The parameter β is commonly referred to as the *coalescence kernel* as it is the mathematical term in the balances that links the physical and process parameters with the

statistical chance of successful permanent coalescence (Ding et al., 2006). In other words, the coalescence kernel governs the mathematical description of coalescence as the term $\beta(x', x, t)$ may be seen as a pseudo rate constant for the agglomeration process. The choice of kernel can dramatically affect the rate of coalescence and thereby the shape of the predicted granule size distribution (Cryer, 1999). Figure 1 indicates examples of the effect of the choice of coalescence kernel on the predicted shape of the final granule size distribution for three simple kernels.

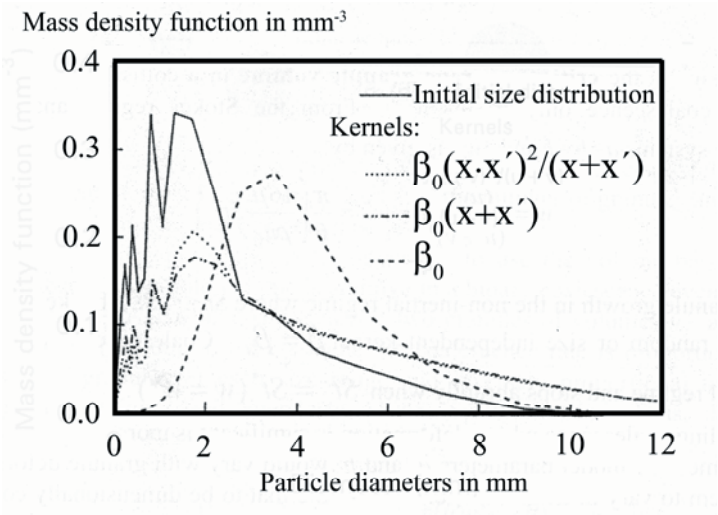

Figure 1: Effects of coalescence kernels.
Effect of the choice of coalescence kernel on the shape of the predicted final granule size distribution (Based on Litster & Ennis, 2004 and Adetayo et al., 1995).

Although some of the modern kernels have a more theoretical foundation, all kernels are empirical or semi-empirical in nature and have been formulation largely upon experimental observations. Kernel expressions are typically a function of the granule size (through the x and x′ dependences) besides a number of fitted coefficients. These empirical coefficients are deduced from curve fitting from experimental data. Historically, much focus has been on the development of new kernels in order to be able to describe agglomeration processes across scale and bed material. One of the first suggestions by Kapur & Fuerstenau in 1969 suggested that β should simply be a constant β_0. Modern considerations by Cameron et al. (2005) and Liu & Litster (2002) indicate that the coalescence kernel is affected by two major factors being first of all the collision probability of the specified pair of particles, and secondly, the chance of successful permanent coalescence or rebound (Cameron et al., 2005). Whereas the first factor mainly depends on the particle sizes, the fluid bed process conditions affecting the particle flow pattern and general operating conditions, Liu et al. (2000) have found, in accordance with the theory of type I and type II coalescence presented in section 1.3.2.2, that the most important aspects affecting the success of coalescence are elastic-plastic properties of the particles, viscous fluid layer, head on collision forces and the energy balance of the system. This has lead to a renewal of the general approach to subdivide the kernel expression into two parts as suggested originally by Sastry in 1975 according to:

$$\beta(x, x') = \beta_0 \cdot \beta^*(x, x') \tag{1.34}$$

where β_0 is the rate constant depending on the operating conditions of the fluid bed. The term $\beta^*(x, x')$ expresses the dependency of the kernel on the size of the coalescing granules. It determines the shape of the resultant granule size distribution (Liu and Litster, 2002). Due to the complexity and limited knowledge of the forces affecting the particles inside the fluid bed, the

form of the coalescence kernel based on physical properties of the particle materials is not yet fully established. However, many empirical and theoretical expressions for the term $\beta^*(x, x')$ have been proposed in the literature, although these expressions generally have been sparsely validated. Table 1 gives a historical summary of the most cited coalescence kernels through the last forty years of population balance modelling of agglomeration.

Kernel	Reference
$\beta = \beta_0$	Kapur and Fuerstanau (1969)
$\beta = \beta_0 \frac{(x + x')^a}{(x \cdot x')^b}$	Kapur (1972)
$\beta = \beta_0 \cdot \beta^*(x, x')$	Sastry (1975)
$\beta = \begin{cases} \beta_0 , t < t_{switch} \\ \beta_1(x, x') , t > t_{switch} \end{cases}$ where β_0 and β_1 are constants and t_{switch} is the time required to reach the final equilibrium size distribution of the first non-inertial stage of granulation.	Adetayo et al. (1995)
$\beta = \begin{cases} \beta_0 , w < w^* \\ 0 , w > w^* \end{cases}$ where $w = \frac{(x \cdot x')^{a_{AE}}}{(x + x')^{b_{AE}}}$ and a_{AE}, b_{AE}, β_0 are constants and w^* the critical granule volume.	Adetayo & Ennis (1997)
$\beta = \beta_0 \int_{-\infty}^{St^*} f(\Phi, t) d\Phi$ where $f(\Phi, t)$ is the discrete probability density function.	Cryer (1999) and Cryer & Scherer (2003)
$\beta = \beta_0 (1/x + 1/x')^{1/2} \cdot (x^{1/3} x'^{1/3})^2$ and $\beta = \beta_0 (1/x^3 + 1/x'^3) \cdot (x^{1/3} x'^{1/3})$	Friedlander (2000)
$\beta\big\|_{x,x'} = \begin{cases} \beta_1 : \textit{for type I and type II coalescence with no permanent deformation} \\ \beta_2 : \textit{for type II coalescence with permanent deformation} \\ 0 : \textit{for rebound} \end{cases}$	Liu & Litster (2002)
$\beta = \beta_0 \cdot (x^3 + x'^3) \left((c_x + c_{x'})^{\alpha_M} \left(100 - \frac{c_x + c_{x'}}{2} \right)^{\delta_M} \right)^{\alpha_M}$ where c_x and $c_{x'}$ represent the volume percentage of binding agent in the agglomerate x and x′ respectively, and α_M and δ_M are fitted parameters.	Madec et al. (2003)
$\beta = \beta_0 \cdot (x, x')^2 \sqrt{\frac{1}{x^3} + \frac{1}{x'^3}}$	Tan et al. (2002 and 2004)

Table 1: Historical summary of proposed coalescence kernels in literature.

In recent years, many theoretical models have been developed to predict the coalescence probability from process conditions and physical properties of the granules and binder materials

as it was also presented in the chapter two. The methods used in developing the models are either energy or force balances and most of the models are able to predict whether the granules will stick together and rebound upon collision. A constant kernel may then be applied to the granules, which successfully coalesce. Since the granule size, granule physical properties, binder properties as well as the collision velocities of the granules are included in these models, the coalescence kernels based on these theoretical models should be more fundamentally sound than the pure empirical kernels (Liu & Litster, 2002). An example of such models is the kernel suggested by Adetayo & Ennis (1997). They suggested a kernel based on the viscous Stokes theory according to[viii]:

$$\beta(x,x') = \begin{cases} \beta_0 \,, w < w^* \\ 0 \,, w \geq w^* \end{cases} \tag{1.35}$$

where w^* is the critical average granule volume and w is defined as:

$$w = \frac{(x \cdot x')^{a_{AE}}}{(x + x')^{b_{AE}}} \tag{1.36}$$

In this approach, collisions lead to successful coalescence only if $w < w^*$ (Rhodes, 1998). The parameters a_{AE} and b_{AE} are constants depending on the deformability of the colliding granules[ix]. In the case of non-deformable granules $a_{AE} = b_{AE} = 1$ and assuming spherical granules, w^* may be expressed as:

$$w = \frac{(x \cdot x')}{(x + x')} = w^* = \frac{\pi}{6}\left(\frac{16 \cdot \eta_{liq}}{\rho_g \cdot u_0} \cdot St^*\right)^3 \tag{1.37}$$

For granule growth in the non-inertial regime where $St << St^*$, this kernel reduces to the simple size independent kernel $\beta = \beta_0$. According to the viscous Stokes theory, coalescence occurs only in the non-inertial regime and stops when $St = St^*$, which by the Adetayo et al. (1997) analogy would be the case when w equals w^*.

Although initial approaches by Adetayo & Ennis (1997) and Liu & Litster (2002), there is generally very limited work on applying the theoretical coalescence models to the population balance equations in the literature. According to Liu & Litster (2002), the reasons for limited work on applying theoretical coalescence models to the population balance modelling of granule size distributions can be attributed by two factors. Firstly, most models are based on the collision mechanisms of two granules. In a fluid bed in which many granules interact with each other the theoretical model based on binary collisions may not be applied with much chance of success. Secondly, there is still very limited knowledge of the granule collision velocity distribution and collision frequencies in fluid beds, meaning that the collision velocity u_0 (or rather the distribution of collision velocities) is extremely difficult to estimate as previously emphasised. Therefore using a kernel or combination of kernels that provide the best fit to experimental data is still the most commonly used method (Liu & Litster, 2002).

1.2.4 About one-dimensional balances

By far, the largest amount of literature regarding population balance modelling of batch granulation systems has used the one-dimensional population balance model as derived in the previous sections. These one-dimensional models have traditionally assumed that granule "size" (usually mass or volume) is the only independent granule property that significantly affects the granule growth behaviour, and that the agglomeration process can be modelled with size as the only internal coordinate completely neglecting the influence of any external coordinates as well as environmental effects. However, several other independent granule properties have been identified and for some time known to strongly influence agglomeration (Iveson, 2002). These include internal coordinates as granule binder content, porosity and primary particle sizedistributions and composition.

The implicit assumption that conditions are spacially uniform throughout the batch granulator does not hold true for many types of fluid beds as segregation has the potential to occur as well as a possible influence of the significant variations in relative humidity and temperature inside the fluid beds, as it has been proved by Maronga & Wnukowski (1997a, 1997b and 1998). This

will affect the frequency and velocity of collisions besides influence the chance of permanent coalescence. Hence, from a formal point of view, the one-dimensional balances are inadequate to completely model the situation during fluid bed granulation. Indeed the problems with coalescence kernel approximations may be seen as an indication of this inadequacy. A reasonable argument is that even with a good fit for the kernel expression there is no guarantee that it is the best fit or that it has any physical basis (Iveson, 2002). In addition, these fits cannot be used for predicting agglomeration behaviour outside the range of conditions in which they were fitted. It is worth noting that even with the most theoretically founded models, as the case with the presented model by Adetayo & Ennis (1997), some characteristic measure of the granule environment is needed being e.g. the granule collision velocity and granule contact time as well as the mechanical properties of the granules being e.g. the coefficients of restitution or plastic yield stress etc. This strongly calls for an expansion of the one-dimensional balances into three or four dimensions in order to include external as well as additional internal coordinates as granule porosity and binder contents in addition to size (Iveson, 2002).

There are however, many reasons for continuing with the simple one-dimensional balances and still articles are being published including the previously derived one-dimensional models. As it will become clear from the following sections, the solution of even the simplest one-dimensional models requires quite a lot of ingenuity and numerical solution techniques. At the present state it is still necessary with a lot of simplifications in order for the simplest models to be solved. Secondly, there is a general lack of systematic data material meaning that validation of the suggested expressions and models almost never happens extensively. New approaches often use older previously reported data to build new and better models and as it is somewhat academically easier to expand previous models than to set up and solve completely new, the trend in population balance modelling continues to apply one-dimensional models. Despite these facts, multi-dimensional models are nevertheless being suggested, as it will be briefly introduced in the following section.

1.3 Multi-dimensional population balances

The idea of extending the general one-dimensional population balances with additional dimensions to describe particulate processes in more detail is not new. In aerosol science, multi-dimensional population balances have been both proposed and somewhat validated (Xiong & Pratsinis, 1993, Verkoeijen et al., 2002 and Wauters, 2001). Xiong & Pratsinis (1993) derived a two-dimensional model to describe the simultaneous coagulation and sintering that occur during a gas phase powder production process. Using particle volume and particle surface area as particle dimensions they were able to describe the change of the particle morphology as well as the average particle size and polydispersibility. Other examples of multi-dimensional population balances have been reviewed by Verkoeijen et al. (2002). In recent years a number of multi-dimensional population balances for batch granulation have been proposed. Cameron et al. (2005) present a two-dimensional balance with two internal coordinates being particle total volume v and the liquid binder volume v_L. As with the previous simple models, perfect mixing is assumed thereby not including any spatial external coordinate in the model. In addition, birth

and death due to breakage is neglected. The proposed 2-D population balance model for a batch granulation process being analogous to the previously derived equation 1.23 is:

$$\underbrace{\frac{\partial f_{bi}(v,v_L,t)}{\partial t}}_{\text{Average number density}} + \underbrace{\frac{\partial}{\partial v}(G(v,t)\cdot f_{bi}(v,v_L,t))}_{\text{Growth due to coating (layering)}} + \underbrace{\frac{\partial}{\partial v_L}(G(v_L,t)\cdot f_{bi}(v,v_L,t))}_{\text{Growth due to coating (layering)}} =$$

$$\underbrace{\frac{1}{2}\int_0^v \int_0^{\min(v_L,\,v-v')} \beta(v-v',v_L-v_L')\cdot f_{bi}(v-v',v_L-v_L',t)\cdot f_{bi}(v',v_L',t)dv_L'dv'}_{\text{Birth due to aggregation}} \qquad (1.38)$$

$$\underbrace{-f_{bi}(v,v_L,t)\int_0^\infty \int_0^{v_L} \beta(v,v_L,v',v_L')\cdot f_{bi}(v',v_L',t)dv_L'dv'}_{\text{Death due to aggregation}}$$

in which the relationship between the bi-variant average number density function f_{bi} and the previously used single-variant average number f_1 is determined according to (Cameron et al., 2005):

$$f_1(v,t) = \int_0^v f_{bi}(v,v_L,t)dv_L \qquad (1.39)$$

Under the assumption that at a given size, all granules have the same liquid content, the two-dimensional population balance equation in equation 1.38 can be reduced to a set of two, one-dimensional balances which makes the solutions somewhat easier. So far this technique has only been applied to high-shear mixing (Cameron et al., 2005, Hounslow et al., 2001 and Biggs et al., 2003).

As a further expansion of the two-dimensional population balance models, Iveson (2002) suggested a four-dimensional population balance model for batch granulation of a binary mixture of solids *a* and b^x. The four independent granule characteristics that were considered were *m*, the granule solid phase, *w*, the binder to solid mass ratio, ε, the granule porosity and *x*, the solid phase mass fraction of component *a*. Iveson (2002) chose granule mass rather than volume as an indicator of granule size as the volume of the granules vary as they consolidate, which nevertheless is a phenomenon often neglected in fluid bed modelling situations as emphasised in Hede (2005). For a batch granulation system in which layering as well as birth and death due to breakage is neglected, Iveson (2002) suggested the following four-dimensional balance according to:

$$\underbrace{\frac{\partial f_{tetra}(m,\varepsilon,w,x,t)}{\partial t}}_{\text{Average number density}} =$$

$$\underbrace{\frac{1}{2}\int_{m_1=0}^{m}\int_{w_1=0}^{\infty}\int_{\varepsilon_1=0}^{1}\int_{x_1=0}^{1}\beta(m_1,m_2,\varepsilon_1,\varepsilon_2,w_1,w_2,x_1,x_2)\cdot}_{\text{Birth due to aggregation, part 1}}$$

$$\underbrace{f_{tetra}(m_1,\varepsilon_1,w_1,x_1,t)\cdot f_{tetra}(m_2,\varepsilon_2,w_2,x_2,t)dm_1d\varepsilon_1dw_1dx_1}_{\text{Birth due to aggregation, part 2}} \qquad (1.40)$$

$$\underbrace{-f_{tetra}(m,\varepsilon,w,x,t)\int_{m_1=0}^{m}\int_{w_1=0}^{\infty}\int_{\varepsilon_1=0}^{1}\int_{x_1=0}^{1}\beta(m_1,m,\varepsilon_1,\varepsilon,w_1,w,x_1,x)\cdot}_{\text{Death due to aggregation, part 1}}$$

$$\underbrace{f_{tetra}(m_1,\varepsilon_1,w_1,x_1,t)dm_1d\varepsilon_1dw_1dx_1}_{\text{Death due to aggregation, part 2}}$$

The coalescence terms in equation 1.40 are calculated by considering all the possible coalescence events that can form a granule with properties: m, ε, w, x. Given the first granule with properties m_1, ε_1, w_1, x_1 the necessary mass, binder content and composition of the second granule can be calculated from conservation of mass equations according to:

$$m_2 = m - m_1 \qquad (1.41)$$

$$m_2 = \frac{w \cdot m - w_1 \cdot m_1}{m_2} = \frac{w \cdot m - w_1 \cdot m_1}{m - m_1} \tag{1.42}$$

$$x_2 = \frac{x \cdot m - x_1 \cdot m_1}{m_2} = \frac{x \cdot m - x_1 \cdot m_1}{m - m_1} \tag{1.43}$$

The porosity of the second granule ε_2 can be calculated by assuming that the total pore volume of the coalesced particles is conserved according to:

$$\left(\frac{\varepsilon_1}{1-\varepsilon_1}\right)\cdot\left(\frac{m_1 \cdot x_1}{\rho_a} + \frac{m_1(1-x_1)}{\rho_b}\right) + \left(\frac{\varepsilon_2}{1-\varepsilon_2}\right)\cdot\left(\frac{m_2 \cdot x_2}{\rho_a} + \frac{m_2(1-x_2)}{\rho_b}\right) = \left(\frac{\varepsilon}{1-\varepsilon}\right)\cdot\left(\frac{m \cdot x}{\rho_a} + \frac{m(1-x)}{\rho_b}\right) \tag{1.44}$$

These four equations assume that the agglomerates instantly homogenise and become spherical after coalescence. In reality of course, the agglomerate dumbbell will take a finite time to spheronise. This is however a common assumption of negligible error (Iveson, 2002). Equation 1.44 additionally assumes that the coalescence event does not cause any change in the total pore volume. However, if permanent plastic deformation occurs, then it is possible for the pore volume to either increase or decrease depending on whether there is a significant amount of dilation or compaction (Iveson, 2002). In the case of fluid bed granulation, compaction and consolidation is often neglected but for high-shear mixing, additional terms accounting for the consolidation can be added to equation 1.44.

Besides the presented balances, a number of other multi-dimensional population balances equations have been presented e.g. by Wauters, 2001 and Immanuel & Doyle (2005). Likewise have Verkoeijen et al. (2002) extended the Iveson (2002) proposal in that they suggest the use of truly mutually independent particle properties as the internal coordinates. Thus, in a three-dimensional formulation they propose the use of the volume of solid, volume of liquid and volume of gas as the internal coordinates rather than the particle total volume, binder content and porosity which are not mutually independent of each other. This approach results in a fortunate separation of the underlying meso-scopic processes of agglomeration, breakage, drying and coating etc. (Verkoeijen et al., 2002 and Cameron et al., 2005). A detailed review of the latest advances in multi-dimensional population balances may be found in Cameron et al. (2005).

Even though multi-dimensional population balances seems promising towards the precise mathematical description of the granulation process, currently no numerical methods for solving the multi-dimensional balances exist. Not only are the multi-dimensional integro differential equations tedious to solve but as the effects that is incorporated into the models are not yet fully understood, the modelling of such phenomena is clearly associated with a large degree of

uncertainty. Although daunting, these challenges cannot be ignored if population balances should be applied for prediction of the various complex phenomena that occurs during granulation, and it seems vital for the use of population balances in the process of scaling granulating systems. As long as the problems of solving multi-dimensional population balances exist it is very likely that people will continue to refine the one-dimensional balances. These simple balances does not to the same extent struggle will inadequate numerical solution techniques, and as it will be presented in the following sections, several approaches may be applied for the solution of one-dimensional population balances.

1.4 Solving one-dimensional population balances

Even in the simple case with one-dimensional population balances, the resulting equation is a partial integro differential equation, which cannot be solved as simple as many of the well-known trivial partial differential equations. Complete analytical solutions are available only for a limited number of special cases (Litster & Ennis, 2004). Fortunately for batch fluid beds, analytical solutions are available for situations in which only either only coating (layering) or agglomeration occurs. These two simple cases will be briefly presented in the following sections.

1.4.1 Analytical solution to the pure growth by coating (layering) situation

In systems where only coating occurs, the derived equation 1.32 turns into the following equation:

$$\frac{\partial f_1(x,t)}{\partial t} + \frac{\partial}{\partial x}(G(x,t) \cdot f_1(x,t)) = 0 \tag{1.45}$$

Assuming linear size independent growth meaning that G = constant ≠ G(x), the solution to equation 1.45 is a travelling wave function with the granule size shifting forward in time with the shape of the granule size distribution unaltered thereby expressing the average number density function as (Litster & Ennis, 2004):

$$f_1(x) = f_{initial}(x - \Delta x) \tag{1.46}$$

in which $f_{initial}$ represents the initial size distribution and Δx may be found according to:

$$\Delta x = \int_0^t G \cdot dt \tag{1.47}$$

According to Litster & Ennis (2004), it is often reasonable to assume a linear growth independent of granule size. This implies that each granule has the same exposure to new coating droplets again implying that each granule has equal exposure time in the spraying zone. In that case, the growth rate G can be directly related to the rate of addition of new coating solution and the surface area of the granules in the granulator according to (Liu & Litster, 1993):

$$G(x) = k' \cdot \frac{\pi d_v^{\,2}}{\zeta} = k \cdot x^{2/3} \tag{1.48}$$

in which k′ is a proportional constant, d_v is the equivalent diameter of the particles, ζ is the particle sphericity and k is another proportionality constant. In typical coating situations the density of the coating material is slightly different from the core material and the average particle density will thereby slightly change as the processing time proceeds. An estimate of the proportionality constant k can be obtained at any time by following the mass balance, which states that the mass increase in the bed per unit time is equal to the dry coating material feed rate S_d minus the rate of elutriation E_{elu} (Liu & Litster, 1993):

$$\int_0^\infty f_1(x,t)\cdot k\cdot x^{2/3}dx = S_d - E_{elu} \Rightarrow$$

$$k = \frac{S_d - E_{elu}}{\int_0^\infty f_1(x,t)\cdot x^{2/3}dx} \tag{1.49}$$

The presented approach to apply population balances for pure growth by coating situations has proven useful in spouted fluid bed coating systems (Liu & Litster, 1993 and Litster & Ennis, 2004) although the complete omission of agglomeration most likely will be an inadequate assumption for most industrial applications as agglomeration almost never can be avoided in common fluid bed coating systems.

1.4.2 Analytical solution to the pure growth by agglomeration situation

In the case where agglomeration is the only dominating growth mechanisms, equation 1.32 can be simplifying according to (Litster & Ennis, 2004):

$$\frac{\partial f_1(x,t)}{\partial t} = \frac{1}{2}\int_0^x \beta(x - x', x')\cdot f_1(x - x',t)\cdot f_1(x',t)dx' - f_1(x,t)\int_0^\infty \beta(x, x')\cdot f_1(x',t)dx' \tag{1.50}$$

It is assumed that the coalescence kernel can be expressed according to equation 1.35-1.37 as suggested by Adetayo & Ennis (1997). According to the viscous Stokes theory, on which this kernel is expressed, the primary agglomeration takes place in the non-inertial regime. Here $w < w^*$ for all particle collisions and $\beta(x, x') = \beta_0$ indicates a size independent kernel. For this case it is possible to calculate the moments solution to the population balance based on the total number of particles N_T at time t. According to Adetayo et al. (1995) and Kapur & Fuerstenau (1969), the zeroth moment of the population balance is:

$$\frac{dN_T}{dt} = -½\beta_0 N_T \Rightarrow N_T = N_{T_0}\cdot\exp\left(-\frac{\beta_0 t}{2}\right) \tag{1.51}$$

in which N_{T_0} is the initial total number of particles. In addition, the mean granule size $\bar{r}_g$ has been observed to vary according to (Adetayo et al., 1995):

$$\bar{r}_g = \bar{r}_{g_0}\cdot\exp\left(\frac{\beta_0 t}{6}\right) \tag{1.52}$$

in which $\bar{r}_{g_0}$ is the initial mean granule size prior to agglomeration. Even though analytical solutions seems convenient, such solutions are however only available for specific initial narrow and unimodal size distributions and only in the simplest case of non-inertial growth, as it has

been shown by (Litster & Ennis, 2004). For practical purposes, solution of population balances is done in terms of numerical methods solely.

1.4.3 Numerical solutions to one-dimensional population balances

The previously presented simplifications to the full population balance have been a convenient way to be able to solve the balances analytically. For practical purposes however these simplifications have proven inadequate and hence numerical solution techniques are necessary in order to solve the complete balance in equation 1.32. Numerical methods allow the shape of the granule size distribution to be easily tracked and all the simplifications required for analytical solutions are not needed.

During the last twenty years, a number of different techniques have been suggested for the numerical solution of the one-dimensional population balances. Some of the first techniques involved the discretisation of the integro differential equation thereby breaking the particle size distribution into discrete intervals, and then solve the resulting series of ordinary differential equations as proposed by Batterham et al. (1981) and others (Litster & Ennis, 2004). At the moment, various numerical solution techniques are available and a brief introduction to the most versatile and commonly applied will be given in the following sections.

1.4.3.1 Hounslow's discretisation method and the Litster expansion

Hounslow et al. (1988) developed a relatively simple discretisation method by employing a M-I approach[xi]. They considered a population balance on the form of equation 1.23 using particle volume as the internal coordinate besides assuming only agglomeration and coating terms to be of importance. Because of the identified advantages of length-based models, Hounslow et al. (1988) performed the coordinate transformation to convert the volume-based model into a length-based model. The method is based on geometric discretisation with the following ratios between two successive size intervals:

$$\Omega_H = \frac{L_{i+1}}{L_i} = \sqrt[3]{2} \text{ or } \frac{v_{i+1}}{v_i} = 2 \tag{1.53}$$

in which L denotes the characteristic length of particles, v the volume of the particles and the subscripts i+1 and i denote the two successive size classes. By applying this technique, the population balance in equation 1.23 can be converted into a set of discretised population balance equations in various size intervals. The individual terms accounting for birth and death due to agglomeration and the term accounting for growth by coating may thereby be expressed as (Hounslow et al., 1988):

$$\text{Birth by agglomeration} : \sum_{j=1}^{i-2} 2^{j-i+1} \beta_{i-1,j} N_j N_{i-1} + \frac{1}{2} \beta_{i-1,i-1} N_{i-1}^2 \tag{1.54}$$

$$\text{Death by agglomeration} : N_i \sum_{j=1}^{i-1} 2^{j-i} \beta_{i,j} N_j - N_i \sum_{j=1}^{\infty} \beta_{i,j} N_j \tag{1.55}$$

$$\text{Growth by coating} : \frac{\partial G N_i}{\partial L} = \frac{2G}{(1+\Omega_H) L_i} \left(\frac{\Omega_H}{\Omega_H{}^2 - 1} N_{i-1} + N_i - \frac{\Omega_H}{\Omega_H{}^2 - 1} N_{i+1} \right) \tag{1.56}$$

in which N_i represents the total number of particles of size class i and $\beta_{i,j}$ is the coalescence kernel for two different discretised volume intervals. Newer more advanced models expand the discretisation for the two agglomeration terms. E.g. have Litster et al. (1995) made an improved discretisation by[xii]:

$$\frac{v_{i+1}}{v_i} = 2^{1/q} \quad , \; q \in \mathbf{N} \tag{1.57}$$

This expands the original Hounslow model to (Litster et al., 1995):

$$\text{Birth by agglomeration}: \sum_{j=1}^{i-S(q)-1} \beta_{i-1,j} N_j N_{i-1} \left\{ \frac{2^{(j-i+1)/q}}{2^{1/q}-1} \right\} + \sum_{k=2}^{q} \sum_{j=i-S(q-k+2)-k+1}^{i-S(q-k+1)-k} \beta_{i-k,j} N_{i-k} N_j \cdot$$

$$\left\{ \frac{2^{(j-i+1)/q} - 1 + 2^{-(k-1)/q}}{2^{1/q}-1} \right\} + \frac{1}{2} \beta_{i-q,i-q} N_{i-q}^2 + \tag{1.58}$$

$$\sum_{k=2}^{q} \sum_{j=i-S(q-k+2)-k+2}^{i-S(q-k+1)-k+1} \beta_{i-k+1,j} N_{i-k+1} N_j \left\{ \frac{-2^{(j-i)/q} + 2^{1/q} - 2^{-(k-1)/q}}{2^{1/q}-1} \right\}$$

$$\text{Death by agglomeration}: \sum_{j=1}^{i-S(q)} \beta_{i,j} N_i N_j \left\{ \frac{2^{(j-1)/q}}{2^{1/q}-1} \right\} - \sum_{j=i-S(q)+1}^{\infty} \beta_{i,j} N_i N_j \tag{1.59}$$

where S(q) is a summation function defined as (Litster et al., 1995):

$$S(q) = \sum_{p=1}^{q} p \tag{1.60}$$

By the expansion by Litster et al. (1995), accuracy is increased by increasing the value of q at the expense of computational time. Given the uncertainty with any practical particle size measurements, Litster & Ennis (2004) suggest that q should be chosen in the range of one to four. An example of the increased accuracy with increasing value of q can be seen from figure 2:

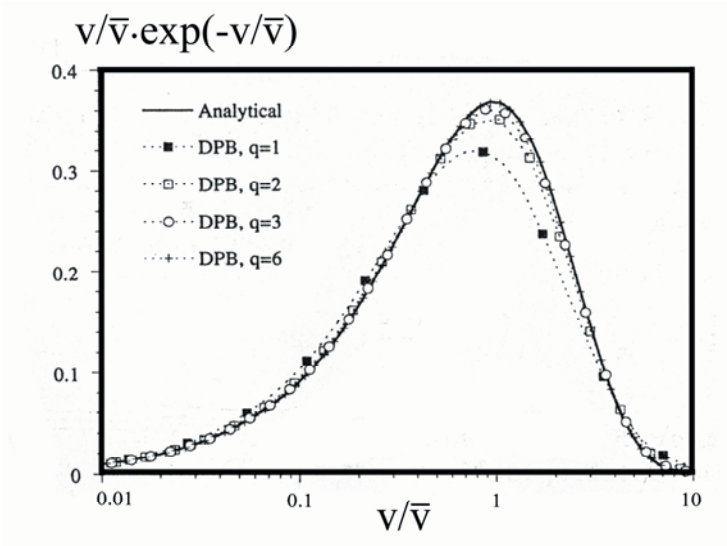

Figure 2: Numerical solution versus analytical.
Comparison of numerical solution of the one-dimensional population balance with a known analytical solution by applying the discretisation refinement suggested by Litster et al. (1995) (based on Litster et al., 1995 and Litster & Ennis, 2004).

Although being fairly simple, the Hounslow/Litster discretisation technique has been widely cited and used for a number of population balance problems besides the ones encountered in granulation systems. Two major advantages associated with the Hounslow/Litster method are

first of all that it is easy to understand and simple to implement in programmes. Secondly, it allows a crude discretisation with a small number of size classes. This second advantage is particular important in order to apply the population balances in process optimisation (Wang et al., 2006). However, there are two significant limitations with this well accepted method. The method does first of all lead to significant errors in large size classes and secondly, the treatment for growth by coating as suggested in equation 1.56 has proven to be too simple in order to avoid numerical dispersion. Fortunately, the reliability of the simulation results does not deteriorate significantly because of this. This is because in the particular processes studied so far, the mass of over-sized particles only consists of small portions in the total mass, and compared with the coalescence growth, the growth by coating only plays a minor role. If however, the Hounslow/Litster method should be applied for primary coating studies these issues have to be taken into account and the Hounslow/Litster method should be replaced by more accurate numerical schemes (Wang et al., 2006).

1.4.3.2 Other methods for numerical solution of population balances

In addition to the Hounslow/Litster approach, a number of other solution techniques have been proposed within the last ten years. E.g. have Kumar and Ramkrishna (1996) developed a discretisation method using a grid with a more general and flexible pattern with fine or coarse discretisation in the different size ranges. This allows a much more refined discretisation in some specifically chosen size ranges while a broader discretisation may be chosen in other size classes of less relevance.

This discretisation technique may however preferably be applied with volume-based population balance models rather than length-based such as the Hounslow/Litster approach (Cameron et al., 2005).

A recent method suggested by Liu & Cameron (2001) is the wavelet-based methods for solving population balance equations being somewhat similar to other collocation methods by normalising the coordinates between zero and one. Wavelet transformation involves representing general functions in terms of simple, fixed building blocks at different scales and positions. These building blocks, which are actually a family of wavelets, are generated from a single, fixed function called the *mother wavelet* by translation and dilation (scaling) operations. In contrast to the traditional trigonometric basis functions, which have infinite support, wavelets have compact support, thus being able to approximate a function not by cancellation, but through placement of the right wavelets at appropriate locations. The multi-resolution analysis properties of wavelets make them attractive candidates for representing functions in differential equations which are being solved numerically (Liu & Cameron, 2001). The wavelet technique is very general, powerful and overcomes the crucial problems of numerical diffusion and stability that often characterize some of the other available numerical solution techniques (Cameron et al., 2005).

Besides the mentioned principles, several other conventional numerical solution techniques have been applied for population balances including method of weighted residuals, method of moments, orthogonal collocation, collocation on finite elements, as it has been reviewed by Ramkrishna (1985 & 2000). One of the newest reviews by Cameron et al. (2005) gives a more in-depth review of some of the newest approaches including advanced Monte Carlo methods[xiii] and finite-element discretisation.

1.5 Inverse problems in population balances

A generic issue in the development and application of population balance models is the identification of the model parameters employing appropriate experimental data (Cameron et al., 2005). The population balance equation is based on a number balance that arises from the consideration of single particle or binary particle interaction behaviour. As the particle behaviour must be considered in the local population setting, it is often not an experimentally accessible quantity, as it requires observation on specific particles that can be readily obscured by the presence of the numerous neighbour particles in real systems. Consequently, an often applied approach has been one of assuming the validity of single particle behaviour obtained in isolation from its neighbours either from specified experiments or simple from theoretical considerations (Ramkrishna, 2000 and Iveson et al., 2001a). Obviously, in order to obtain better characterisations of single particle behaviour in a population, experimental observations must be made on the population and a method must be found to extract the behaviour of single particles from such measurements (Ramkrishna, 2000). Such approaches and techniques are commonly referred to as the *Inverse Problem* being inverse in the sense that models are deduced from experimental data primarily. It will only be briefly introduced in the present section, as it requires a previous introduction to self-similar behaviour and grey-box modelling strategies,

which are beyond the scope of this review. It should be noted however, that the solution of the inverse problem constitutes a crucial step in the use of population balances for many practical applications. An introductory review may be found in Cameron et al. (2005) and Ramkrishna & Mahoney (2002), and Ramkrishna (2000) provides a throughout introduction in the field of inverse problems.

In an inverse problem approach by Mahoney et al. (2002), the population balance equation is solved under suitable assumptions by the method of characteristics, which associates the number density for any particle size at any time with a single point from the initial or boundary condition. The key in using this technique is the recognition that these characteristics correspond to the size history of individual particles and can be associated with constant cumulative number densities (quantiles) of the population. These quantiles are easily identifiable from experimental data. The variation of size and number density along these characteristics provides decoupled equations used to determine the growth rate. Validity of the determined growth law is checked by the collapse of the experimental data onto initial and boundary conditions.

A general advantage of inverse problem models is that they are not committed to any specific form of the model under investigation. This means that when a model form turns out to be inappropriate, new parameter-fitting procedures can be readily included in order to build a more adequate model. It may seem advantageous at first to apply inverse problem models in an industrial context but there are however a number of disadvantages. Extracting information from experimental data requires in fact quite a lot of assumptions in order to be used in models. In the case of using experimental data in population balance modelling, it is desirable to consider situations in which only one of the different particle processes occurs so that the inversion can be accomplished without unduly risking loss of uniqueness. This is a difficult task with fluid beds as it is almost impossible to make sure that the particle sample extracted from the bulk has experienced only the one type of particle process (e.g. breakage) that the sample should be used to model. In general, inverse problem models struggles with the problem that even tiny errors in the input data produces large errors in the extracted information. This calls for some presmoothing of the input data for such models thereby substantially raising the required amount of data material (Ramkrishna, 2002). The use of inverse problem models is a fairly new alternative to the traditional population balance approach but the sparsely amount of people working in this field, and thereby the limited available experience in literature, means that the real advantages and possibilities with inverse problem modelling are yet to be seen.

Summary

Basically, all models are wrong although some are useful and may be applied with sufficient accuracy. According to Iveson (2002), a useful model is one that correctly accounts for all the effects of first-order significance whilst ignoring unnecessary complications due to effects of secondary importance. Applying population balances in particle technology is a purely academic field, as it was introduced in the text. Since the introduction of the one-dimensional balances more than forty years ago, the one-dimensional balances have developed to include both growth by coating, birth and death by agglomeration and latest, birth and death by breakage.

As it is the case in many other mathematical modelling fields, the solution of the problems by far exceeds the work required to set up the equations. This is also the case for population balance modelling and much effort has been invested into numerical solution techniques for the one-dimensional balances. Still new article are being published in which new and advanced solution techniques are being suggested for the solution one-dimensional balances. Much of this development has been at the expense of proper model validation and along with the advanced "mathemisation" of the population balances, the field is to some extent moving further and further away from granulation physics and experimental data. This is an obvious disadvantage as results must be extensively validated at this present very early stage of quantitative wet granulation modelling.

With a number of critical articles in recent years it becomes even clearer that the one-dimensional population balances are too simple to account for all of the important granulation phenomena and also that new kernel expressions cannot set these issues right. That does not mean that one-dimensional population balances may not at some point in the future be used as rough guidelines with adequate precision for industrial process control issues, but merely that one-dimensional balance are not and most likely will not be capable of, at any point, to describe the granulating system with a precision satisfactory for scientists. The multi-dimensional population balances seem to have the greatest potential for future modelling of fluid bed granulation processes but as long as no versatile solution techniques are available, it does not make much sense to set up the balances.

At the moment the PB models are used more as a learning tool to understand processes and mechanisms rather than being a final unambiguous optimisation tool. Population balances models have to some extent been used successfully to provide insight into the separate mechanisms by which particles grow. However, since powder characteristics, essential hydrodynamic parameters regarding liquid-solid contacting, particles mixing and agglomeration are lumped into the kinetic rate constants, population balances cannot be applied for a-priori design and scale-up of fluid bed granulation processes (Goldschmidt et al., 2003).

Whereas the general perspective of using population balances for granulation process control looks somewhat promising, the use of population balances for scale-up purposes or for the description of the breakage mechanisms associated with granulation has not been proven useful yet (Faure et al., 2001). Much work is still needed before population balance modelling will achieve any significant commercial interest, and as long as the pure academic modelling perspective keeps expanding at the expense of proper validation, industrial application of population balance theory does not seem within immediate reach.

Table of symbols

Nomenclature		Unit (SI-system)
a	Internal coordinate	-
a´	Material constant	Dimensionless
a_d	Projected area of liquid binder droplets	m^2
a_{AE}	Fitting parameter	Dimensionless
a_n	Projected area of a nucleus granule	m^2
$\dot{A}$	Powder flux	m^2/s
A^*	Contact area between colliding granules	m^2
b	Internal coordinate	-
b_{AE}	Fitting parameter	Dimensionless
c	Cohesivity of dry particle mass	N/m^2
$d_{air\ distrib\ pl.}$	Air distribution plate diameter	m
d_b	Gas bubble diameter	m
d_{bed}	Fluidised bed diameter	m
d_d	Liquid droplet diameter	m
$d_{d,rel}$	Relative liquid droplet diameter	m
d_p	Particle diameter	m
$d_{orifices}$	Pitch orifice diameter	m
$d_{sp/sp}$	Interaction parameter of two spheres	m
d_v	Equivalent diameter of particles	m
d_{vessel}	Fluid bed vessel diameter	m
e	Particle coefficient of restitution	Dimensionless
E	Young modulus	N/m^2
E^*	Granule Young modulus	N/m^2
E_{elu}	Elutriation rate	-
$f_1(\mathbf{x}, \mathbf{r}, t)$	Average number density function	-
f_{bi}	Bi-variant average number density function	-
$f_{initial}$	Initial average number density function	-
f_{tetra}	Tetra-variant average number density function	-
$F_{pend.,bound.}$	Pendular force in the "boundary" method	N
$F_{pend.,eq\ sph.}$	Pendular force between two equally sized spheres	N
$F_{pend.,gorge.}$	Pendular force in the "gorge" method	N
F_{vis}	Viscous force	N
$\mathbf{F}_i$	Net force vector acting on particle i	-
$\mathbf{F}_i^{\mathbf{H}}$	Drag force vector	-

$\mathbf{F}_i^E$	Force vector accounting for external fields	-
$\mathbf{F}_i^P$	Force vector accounting for particle-particle interactions	-
g	Gravity	m/s^2
G(**x**, **r**, t)	Rate of growth by layering	-
G_s	Mass flux of particles	m^2/s
h(**x**, **r**, t)	Net generation rate of particles	-
h_0	Binder layer thickness covering colliding granules	m
h_a^+	Birth of particles due to aggregation	-
h_a^-	Death of particles due to aggregation	-
h_{asp}	Characteristic length scales of surface asperities	m
h_b^+	Birth of particles due to breakage	-
h_b^-	Death of particles due to breakage	-
h_{bed}	Bed height	m
H	Separation distance between two spheres	m

i	Summation parameter	-
I_i	Moment of inertia	-
J	Nucleation ratio	Dimensionless
k	Proportionality constant	-
k´	Proportionality constant	-
k_{cn}	Coordination number	Dimensionless
K_a	Nucleation area ratio	Dimensionless
L	Characteristic length of particles	m
L_{bed}	Fluid bed length from distributor plate to exhaust exit	m
L_{slr}	Length scale ratio	Dimensionless
m	Mass	kg
m(**x**)	Mass of a particle of internal state **x**	
m_{aggl}	Agglomerate mass	kg
m_{bed}	Bed load	kg
m_{harm}	Harmonic mean granule mass	kg
$\dot{m}_{nozzle\ air}$	Spray rate from the nozzle	kg/s
$\dot{m}_{spray}$	Air flow rate through the nozzle	kg/s
$\mathbf{M}_i$	Net torque vector	-
n_{fi}	Flow index	Dimensionless
n(**x**, **r**, t)	Actual number density	-
$\dot{n}_0$	Nucleation rate	No. of particles/s
N(**r**, t)	Average total number of particles	-
N_T	Total number of particles	-
N_{T0}	Initial total number of particles	-
p	Summation number	Dimensionless
P	Pressure	Pa
P(**x**,**r**|**x**´,**r**´)	Probability density function	-
q	Discretisation number	Dimensionless
r	Radius	m
r_{aggl}	Radius of an agglomerate	m
r_{def}^*	Critical radius of an agglomerate after which def. occurs	m
r_{harm}	Harmonic mean granule radius	m

r_{neck}	Pendular bridge neck radius	m
$\bar{r}_g$	Mean granule size	m
$\bar{r}_{g_0}$	Initial mean granule size	m
r_{neck}	Pendular bridge neck radius	m
r	External coordinate vector	-
r´	External coordinate vector	-
R	Radius	m
R_p	Particle radius	m
s_{max}	Maximum pore liquid saturation	Dimensionless
S	Distance	m
S_c	Saturation at transition funicular/capillary state	Dimensionless
S_d	Dry coating material feed rate	-
S_f	Saturation at transition pendular/funicular state	Dimensionless
S_{sat}	Amount of saturation	Dimensionless
St_{def}	Stokes deformation number	Dimensionless
St_{def}^*	Critical Stokes deformation number	Dimensionless
St_v	Viscous Stokes number	Dimensionless
St_v^*	Critical viscous Stokes number	Dimensionless
S_w	Wetting saturation	Dimensionless
S(q)	Summation function	-
$S_{Kolmogorov}$	Kolmogorov entropy	bits/s
t	Time	s
t_{coat}	Coating time	s
u	Granule velocity	m/s
u_0	Initial granule collision velocity	m/s
U	Fluidisation velocity	m/s
U_{br}	Bubble rise velocity for a fluid bed	m/s
U_{mf}	Minimum fluidisation velocity	m/s
U_s	Superficial gas velocity	m/s
$\mathbf{v}_i$	Velocity vector	-
v	Particle volume internal coordinate	-
$\bar{v}$	Average particle volume	m^3
v_L	Liquid binder volume internal coordinate	-
$\dot{V}$	Volumetric spray rate	m^3/s
V_{aggl}	Agglomerate volume	m^3
V_{bridge}	Liquid bridge volume	m^3
$V_{\mathbf{r}}$	Volume of external coordinates	-

$V_{\mathbf{x}}$	Volume of internal coordinates	-
w	Granule volume parameter in coal. kernel expression	-
w^*	Critical average granule volume	-
w_{mr}	Mass ratio of liquid to solid	Dimensionless
W	Spray zone width	m
x	Internal coordinate vector	-
x´	Internal coordinate vector	-
x	Coordinate	m
y	Coordinate along the width of the spray zone	m
Y(**r**,t)	Continuous phase vector	-
Y_d	Plastic yield stress	N/m^2
z	Counting number	Dimensionless

Greek

β	Coalescence kernel	-
β_0	Rate constant	-
β_{dt}	Aggregation probability in time interval dt	-
β_{id}	Coefficient of interphase drag	Dimensionless

β^*	Coalescence kernel expression	-
δ	Dimensionless bubble spacing	Dimensionless
δ_{pdef}	Extent of permanent plastic deformation	Dimensionless
μ_f	Coefficient of internal friction	Dimensionless
μ_{mean}	Mean in the Gaussian distribution	m
σ_f	Macroscopic shear stress at failure	Pa
σ_n	Macroscopic normal stress	Pa
σ_{width}	Standard deviation	m
$\sigma_{t,f}$	Funicular bridge static tensile strength	N/m^2
$\sigma_{t,p}$	Pendular bridge static tensile strength	N/m^2
$\sigma_{t,c}$	Capillary bridge static tensile strength	N/m^2
$\sigma(\dot{\gamma})$	Characteristic stress in an agglomerate	N/m^2
σ_y	Yield stress/strength	N/m^2
τ_c	Average particle circulation time	s
τ_d	Droplet penetration time	s
ψ_a	Dimensionless spray flux	Dimensionless
$\psi_n(y)$	Dimensionless nuclei distribution function	Dimensionless
ψ_n	Dimensionless spray number	Dimensionless
ζ	Particle shape factor (sphericity)	Dimensionless
λ	Dimensionless parameter in the dynamic strength eq.	Dimensionless
φ	Half filling radius	°
Θ	Contact angle	°
ν	Poisson ratio	Dimensionless
$\nu(\mathbf{x}', \mathbf{r}', \mathbf{Y}, t)$	Average number of particles formed from break up	-
ε	Particle voidage (void fraction)	%
$\varepsilon_{longitudinal}$	Longitudinal extension strain	Dimensionless
ε_{min}	Minimum porosity	%
ε_{trans}	Transverse contraction strain	Dimensionless
$\overline{\varepsilon}_g$	Mean granule porosity (void fraction)	%
γ_{lv}	Interfacial surface tension between liquid and vapour	N/m
$\dot{\gamma}$	Shear rate	s^{-1}
ρ	Density	kg/m^3
ρ_b	Binder liquid density	kg/m^3
ρ_g	Granule density	kg/m^3
ρ_p	Particle density	kg/m^3
η_{app}	Apparent viscosity	kg s /m
η_{liq}	Liquid (binder/coating) viscosity	kg s /m
$\boldsymbol{\omega}_i$	Angular velocity vector	-
Ω_H	Hounslow discretisation parameter	-
$\Omega_{\mathbf{r}}$	Domain of external coordinates	Dimensionless
$\Omega_{\mathbf{x}}$	Domain of internal coordinates	Dimensionless

Literature

Abbott, A.: *Boundary Between Coating and Granulation*, Master Thesis, Department of Chemical Engineering, The University of Queensland, **2002**.

Adams, M.J. and Perchard, V.: *The Cohesive Forces between Particles with Interstitial Liquid*, International Chemical Engineering Symposium Series, No. 91, pp. 147-160, **1984**.

Adetayo, A.A., Litster, J.D., Pratsinis, S.E. and Ennis, B.J.: *Population balance modelling of drum granulation of materials with wide size distribution*, Powder Technology, No. 82, pp. 37-49, **1995**.

Adetayo, A.A. and Ennis, B.J.: *Unifying Approach to Modelling Granule Coalescence Mechanisms*, AIChE Journal, No. 43, pp. 927-934, **1997**.

Batterham, R.J., Hall, J.S. and Barton, G.: *Pelletizing Kinetics and Simulation of Full-Scale Balling Circuits*, Proceedings 3rd International Symposium on Agglomeration, Nürnberg, Germany, **1981**.

Beekman, W.J.: *Measurement of the Mechanical Strength of Granules*, Ph.D. Thesis, Technische Universiteit Delft, **2000.**

Bell, T.A.: *Challenges in the scale-up of particulate processes – an industrial perspective*, Powder Technology, No. 150, pp. 60-71, **2005**.

Biggs, C.A., Sanders, C., Scott, A.C., Willemse, A.W. Hoffman, A.C., Instone, T., Salman, A.D. and Hounslow, M.J.: *Coupling granule properties and granulation rates in high-shear granulation*, Powder Technology, No. 130, pp. 162-168, **2003**.

Boerefijn, R. and Hounslow, M.J.: *Studies of fluid bed granulation in an industrial R&D context*, Chemical Engineering Science, No. 60, pp. 3879-3890, **2005**.

Cain, R.G., Page, N.W. and Biggs, S.: *Microscopic and macroscopic effects of surface lubricant films in granular shear*, Physical Review E 62, pp. 8369-8379, **2000**.

Cameron, I.T., Wang, F.Y., Immanuel, C.D. and Stepanek, F.: *Process systems modelling and applications in granulation: A review*, Chemical Engineering Science, pp. 3723-3750, **2005**.

Campbell, C.S. and Brennen, C.E.: *Computer simulation of granular shear flows*, Journal of Fluid Mechanics, No. 151, pp. 167-188, **1985**.

Chiesa, M., Mathisen, V., Melheim, J.A. and Halvorsen, B.: *Numerical simulation of particulate flow by the Eulerian-Lagrangian and the Eulerian-Eulerian approach with application to a fluidized bed*, Computers & Chemical Engineering, No. 29, pp. 291-304, **2005**.

Christensen, G., Both, E. and Sørensen, P.Ø.: *Mekanik*, Department of Physics, Technical University of Denmark, **2000**.

Cooper, S. and Coronella, C.J.: *CFD simulations of particle mixing in a binary fluidized bed*, Powder Technology, No. 151, pp. 27-36, **2005**.

Cryer S.A.: *Modelling Agglomeration Processes in Fluid-Bed Granulation*, AIChE Journal, Vol. 45, No. 10, pp. 2069-2078, **1999**.

Cryer S.A. and Scherer, P.N.: *Observations and Process Parameter Sensitivities in Fluid-Bed Granulation*, AIChE Journal, Vol. 49, No. 11, pp. 2802-2809, **2003**.

Davidson, J.F. and Harrison, D.: *Fluidized Particles*, Cambridge University Press, New York, **1963**.

Depypere, F.: *Characterisation of Fluidised Bed Coating and Microcapsule Quality: A Generic Approach*, Ph.D. Thesis, University of Ghent, **2005**.

Ding, A., Hounslow, M.J. and Biggs, C.A.: *Population balance modelling of activated sludge flocculation: Investigating the size dependence of aggregation, breakage and collision efficiency*, Chemical Engineering Science, No. 61, pp. 63-74, **2006**.

Ellenberger, J. and Krishna, R.: *A Unified Approach to the Scale-up of Gas-Solid Fluidized Bed and Gas-Liquid Bubble Column Reactors*, Chemical Engineering Science, Vol. 49, No. 24B, pp. 5391-5411, **1994**.

Ennis, B.J., Li, J., Tardos, G.I., Pfeffer, R.: *The influence of viscosity on the strength of an axially strained pendular liquid bridge*, Chemical Engineering Science, No. 45, pp. 3071-3088, **1990**.

Ennis, B.J., Tardos, G. and Pfeffer, R.: *A microlevel-based characterization of granulation phenomena*, Powder Technology, No. 65, pp. 257-272, **1991**.

Ennis, B.J. and Sunshine, G.: *On Wear as a mechanism of granule attrition*, Tribology International, Butterworth-Heinemann Ltd., pp. 319-327, **1993**.

Fairbrother, R.J. and Simons, S.J.R.: *Modelling of Binder-Induced Agglomeration*, Particle and Particle Systems Characterization, No. 15, pp. 16-20, **1998**.

Fan, R., Marchisio, D.L. and Fox, R.O.: *Application of the Direct Quadrature Method of Moments to Polydisperse Gas-Solid Fluidized Beds*, Preprint submitted to Elsevier Science, **2003**.

Faure, A., York, P. and Rowe, R.C.: *Process control and scale-up of pharmaceutical wet granulation processes: a review*, European Journal of Pharmaceutics and Biopharmaceutics, No. 52, pp. 269-277, **2001**.

Flemmer, C.L.: *On the regime boundaries of moisture in granular materials*, Powder Technology, No. 66, pp. 191-194, **1991**.

Friedlander, S.K: Smoke, *Dust and Haze. Fundamentals of Aerosol Dynamics*, 2nd Edition, Oxford University Press, **2000**.

Fu, J., Adams, M.J., Reynolds, G.K., Salman, A.D. and Hounslow, M.J.: *Impact deformation and rebound of wet granules*, Powder Technology, No. 140, pp. 248-257, **2004**.

Fu, J., Reynolds, G.K., Adams, M.J., Hounslow, M.J. and Salman, A.D.: *An experimental study of the impact breakage of wet granules*, Chemical Engineering Science, No. 60, pp. 4005-4018, **2005**.

Gantt, J.A. and Gatzke, E.P.: *High shear granulation modelling using a discrete element simulation approach*, Powder Technology, No. 156, pp. 195-212, **2005**.

Gera, D., Gautam, M., Tsuji, Y., Kawaguchi, T. and Tanaka, T.: *Computer simulation of bubbles in large-particle fluidized beds*, Powder Technology, pp. 38-47, **1998**.

Gidaspow, D., Jung, J.W. and Singh, R.K.: *Hydrodynamics of fluidization using kinetic theory: an emerging paradigm*, Powder Technology, No 148, pp. 123-141, **2004**.

Glicksman, L.R.: *Scaling Relationships For Fluidized Beds*, Chemical Engineering Science, Vol. 39, No. 9, pp. 1373-1379, **1984**.

Glicksman, L.R.: *Scaling relationships for fluidized beds*, Chemical Engineering Science, Vol. 43, No. 6, pp. 1419-1421, **1988**.

Glicksman, L.R., Hyre, M. and Woloshun, K.: Simplified scaling relationships for fluidized beds, Powder Technology, No. 77, pp. 177-199, **1993**.

Goodwin, J.: *Colloids and Interfaces with Surfactants and Polymers – An Introduction*, John Wiley & Sons Ltd., Chichester, **2004**.

Goldman, A.J., Cox, R.G. and Brenner, H.: *Slow viscous motion of a sphere parallel to a plane wall: I. Motion through a quiescent fluid*, Chemical Engineering Science, No. 22, pp. 637-651, **1987**.

Goldschmidt, M.J.V.: *Hydrodynamic Modelling of Fluidised Spray Granulation*, Ph.D. Thesis, University of Twente, **2001**.

Goldschmidt, M.J.V., Weijers, G.G.C., Boerefijn, R. and Kuipers, J.A.M.: *Discrete element modelling of fluidised bed spray granulation*, Powder Technology, No. 138, pp. 39-45, **2003**.

Goldschmidt, M.J.V., Beetstra, R. and Kuipers, J.A.M.: *Hydrodynamic modelling of dense gas-fluidised beds: comparison and validation of 3D discrete particle and continuum models*, Powder Technology, No. 142, pp. 23-47, **2004**.

Hapgood, K.: *Nucleation and binder dispersion in wet granulation*, Ph.d. Thesis, University of Queensland, **2000**.

Hapgood, K.P., Litster, J.D., Smith, R.: *Nucleation Regime Map for Liquid Bound Granules*, AIChE Journal, No. 49, pp. 350-361, **2003**.

Hapgood, K.P., Litster, J.D., White, E.T., Mort, P.R. and Jones, D.G.: *Dimensionless spray flux in wet granulation: Monte-Carlo simulations and experimental validation*, Powder Technology, No. 14, pp. 20-30, **2004**.

Hede, P.D.: *Fluid bed coating and granulation*, Master Thesis, Department of Chemical Engineering, Technical University of Denmark, **2005**.

Hede, P.D.: *Towards Mathesis Universalis: Modern aspects of modelling batch fluid bed agglomeration and coating systems - review*, Department of Chemical Engineering, Technical University of Denmark, pp. 1-100, **2006**.

Hede, P.D.: *Fluid Bed Particle Processing*, ISBN 87-7681-153-0, Ventus Publishing, pp. 1 -80, **2006**.

Hjortsø, M.A.: *Population Balances in Biomedical Engineering – Segregation through the Distribution of Cell States*, McGraw-Hill , NY, **2006**.

Hoomans B.P.B., Kuipers J.A.M., Briels W.J. and Van Swaaij W.P.M.: *Discrete particle simulation of bubble and slug formation in a two-dimensional gas-fluidised bed: a hard sphere approach*, Chemical Engineering Science., No. 51, pp. 99-118, **1996**.

Hoomans B.P.B.: *Granular dynamics of gas-solid two-phase flows*, Ph.D. Thesis, University of Twente, **1999**.

Hoomans, B.P.B., Kuipers J.A.M. and Van Swaaij W.P.M.: *Granular dynamics simulation of segregation phenomena in bubbling gas-fluidised beds*, Powder Technology, No. 109, pp. 41-48, **2000**.

Horio, M., Nonaka, A., Sawa, Y. and Muchi, I.: *A New Similarity Rule for Fluidized Bed Scale-up*, AIChE Journal, Vol. 32, No. 9, pp. 1466-1482, **1986**.

Hotta, K., Takeda, K. and Iinoya, K.: *The Capillary Binding Force of a Liquid Bridge, Powder Technology*, No. 10, pp. 231-242, **1974**.

Hounslow, M.J., Ryall, R.L. and Marshall, V.R.: *A Discretized Population Balance for Nucleation, Growth and Aggregation*, AIChE Journal, No. 34, pp. 1821-1832, **1988**.

Hounslow, M.J., Pearson, J.M.K. and Instone, T.: *Tracer Studies of high-shear granulation: II. Population balance modelling*, AIChE Journal, No. 47, pp. 1984-1999, **2001**.

Hulburt, H.M. and Katz, S.: *Some problems in particle technology – A statistical mechanical formulation*, Chemical Engineering Science, No. 19, pp. 555-574, **1964**.

Immanuel, C.D. and Doyle, F.J.: *Solution technique for a multi-dimensional population balance model describing granulation processes*, Powder Technology, No. 156, pp. 213-225, **2005**.

Israelachvili, J.N.: *Intermolecular and Surface Forces*, 2nd Edition, Academic Press, San Diego, **1992**.

Iveson, S.M., Litster, D.L.: *Growth regime map for liquid-bound granules*, AIChE Journal, No. 44, pp. 1510-1518, **1998**.

Iveson, S.M., Litster, D.L., Hapgood, K. and Ennis, B.J.: *Nucleation, growth and breakage phenomena in agitated wet granulation processes: a review*, Powder Technology, No. 117, pp. 3-39, **2001a**.

Iveson, S.M., Wauters, P.A.L., Forrest, S., Litster, D.L., Meesters, G.M.H. and Scarlett, B.: *Growth regime map for liquid-bound granules: further development and experimental validation*, Powder Technology, No. 117, pp. 83-97, **2001b**.

Iveson, S.M.: *Granule coalescence modelling: including the effects of bond strengthening and distributed impact separation forces*, Chemical Engineering Science, No. 56, pp. 2215-2220, **2001**.

Iveson, S.M.: *Limitations of one-dimensional population balance models of wet granulation processes*, Powder Technology, No. 124, pp. 219-229, **2002**.

Iveson, S.M., Beathe, J.A. and Page, N.W.: *The dynamic strength of partially saturated powder compacts: the effect of liquid properties*, Powder Technology, No. 127, pp. 149-161, **2002**.

Jain, K.: *Discrete Characterization of Cohesion in Gas-Solid Flows*, Master Thesis, School of Engineering, University of Pittsburgh, **2002**.

Jen, C.O. and Tsao, K.C.: *Coal-Ash Agglomeration Mechanism and its Application in High Temperature Cyclones*, Separation Science and Technology, No. 15, pp. 263-276, **1980**.

Kapur, P.C. and Fuerstenau, D.W.: *A Coalescence Model for Granulation*, Industrial & Engineering Chemistry Process Design Development, No. 8, pp. 56-62, **1969**.

Kapur, P.C.: *Kinetics of granulation by non-random coalescence mechanism*, Chemical Engineering Science, No. 27, pp. 1863-1869, **1972**.

Kaye, B. H.: *Powder Mixing*, Powder Technology Series, Chapman & Hall, London, **1997**.

Kerkhof, P.J.A.M.: *Some modelling aspects of (batch) fluid-bed drying of lifescience products*, Chemical Engineering and Processing, No. 39, pp. 69-80, **2000**.

Khan, I and Tardos, G.I.: *Stability of wet agglomerates in granular shear flows*, Journal of Fluid Mechanics, No. 347, pp. 347-368, **1997**.

Knowlton, T.M., Karri, S.B.R. and Issangya, A.: *Scale-up of fluidized-bed hydrodynamics*, Powder Technology, No. 158, pp. 72-77, **2005**.

Krishna, R. and van Baten, J.M.: *Using CFD for scaling up gas-solid bubbling fluidised bed reactors with Geldart A powders*, Chemical Engineering Science, No. 82, pp. 247-257, **2001**.

Kumar, S and Ramkrishna, D.: *On the Solutions of Population Balance Equations by Discretisation – II. A Moving Pivot technique*, Chemical Engineering Science, No. 51, pp. 1333-1342, **1996**.

Kunii, D. and Levenspiel, O.: *Fluidization Engineering*, 2nd Edition, Butterworth-Heinemann, Stoneham, **1991**.

Lettieri, P, Cammarata, L., Micale, G.D.M. and Yates, J.: *CFD simulations of gas fluidized beds using alternative Eulerian-Eulerian modelling approaches*, International Journal of Chemical Reactor Engineering, No. 1, pp. 1-19, **2003**.

Leuenberger, H.: *Scale-up of granulation processes with reference to process monitoring*, Acta Pharmaceutical Technology, No. 29, pp. 274-280, **1983**.

Leuenberger, H.: *Scale-up in the 4th dimension in the field of granulation and drying or how to avoid classical scale-up*, Powder Technology, No. 130, pp. 225-230, **2003**.

Lian, G., Thornton, C and Adams, M.J.: *A Theoretical Study of the Liquid Bridge Forces between Two Rigid Spherical Bodies*, Journal of Colloid and Interface Science, No. 161, pp. 138-147, **1993**.

Lian, G., Thornton, C and Adams, M.J.: *Discrete particle simulation of agglomerate impact coalescence*, Chemical Engineering Science, No. 19, pp. 3381-3391, **1998**.

Litster, J.D., Smit, J. and Hounslow, M.J.: *Adjustable Discretized Population Balance for Growth and Aggregation*, AIChE Journal, Vol. 41, No. 3, pp. 591-603, **1995**.

Litster, J.D., Hapgood, K.P., Michaels, J.N., Sims, A., Roberts, M., Kameneni, S.K. and Hsu, T.: *Liquid distribution in wet granulation: dimensionless spray flux*, Powder Technology, No. 114, pp. 32-39, **2001**.

Litster, J.D., Hapgood, K.P., Michaels, J.N., Sims, A., Roberts, M. and Kameneni, S.K.: *Scale-up of mixer granulators for effective liquid distribution*, Powder Technology, No. 124, pp. 272-280, **2002**.

Litster, J.D.: *Scaleup of wet granulation processes: Science not art*, Powder Technology, No. 130, pp. 35-40, **2003**.

Litster, J.D. and Ennis, B.: *The Science and Engineering of Granulation Processes*, Kluwer Academic Publishers, Dordrecht, **2004**.

Liu Y. and Cameron, I.T.: *A new wavelet-based method for the solution of the population balance equation*, Chemical Engineering Science, No. 56, pp. 5283-5294, **2001**.

Liu, L.X. and Litster, J.D.: *Coating mass distribution from a spouted bed seed coater: experimental and modelling studies*, Powder Tchnology, No. 74, pp. 259-270, **1993**.

Liu, L.X., Litster, J.D., Iveson, S.M. and Ennis, B.J.: *Coalescence of Deformable Granules in Wet Granulation Processes*, AIChE Journal, Vol. 46, No. 3, pp. 529-539, **2000**.

Liu, L.X. and Litster, J.D.: *Population balance modelling of granulation with a physically based coalescence kernel*, Chemical Engineering Science, No. 57, pp. 2183-2191, **2002**.

London. Imperial College London. Visit on the homepage: www.imperial.ac.uk, December **2005**.

Lödige. *High-shear scaling-up meeting at Novozymes A/S by Horst Spittka from Lödige Process Technology*, Novozymes A/S Bagsværd, 17th of January **2006**.

Madec, L., Falk, L. and Plasari, E.: *Modelling of the agglomeration in suspension process with multi-dimensional kernels*, Powder Technology, No. 130, pp. 147-153, **2003**.

Mahecha-Botero, A., Elnashaie, S.S.E.H., Grace, J.R. and Jim-Lim, C.: *FEMLAB simulations using a comprehensive model for gas fluidized-bed reactors*, Comsol Multiphysics, **2005**.

Mahoney, A.W., Doyle F.J. and Ramkrishna, D.: *Inverse Problems in Population Balances: Growth and Nucleation from Dynamic Data*, AIChE Journal, No. 48, pp. 981-990, **2002**.

Maronga, S.J. and Wnukowski, P.: *Establishing temperature and humidity profiles in fluidized bed particulate coating*, Powder Technology, No. 94, pp. 181-185, **1997a**.

Maronga, S.J. and Wnukowski, P.: *Modelling of the three-domain fluidized-bed particulate coating process*, Chemical Engineering Science, No. 17, pp. 2915-2925, **1997b**.

Maronga, S.J. and Wnukowski, P.: *The use of humidity and temperature profiles in optimising the size of fluidized bed in a coating process*, Chemical Engineering and Processing, No. 37, pp. 423-432, **1998**.

Mazzone, D.N., Tardos, G.I. and Pfeffer, R.: *The behaviour of liquid bridges between two relatively moving particles*, Powder Technology, No. 51, pp. 71-83, **1987**.

Mehta, A.M.: *Scale-up considerations in the fluid-bed process for controlled-release products*, Pharmaceutical Technology, No. 12, pp. 46-52, **1988**.

Merrow, E.W.: *Problems and progress in particle processing*, Development Technology, Chemical Innovation, **2000**.

Michaels, J.N.: *Toward rational design of powder processes*, Powder Technology, No. 138, pp. 1-6, **2003**.

Mort, P.R.: *A multi-scale approach to modelling and simulation of particle formation and handling processes*, Proceedings of the 4th International Conference for Conveying and Handling of Particulate Solids, Budapest, Hungary, **2003**.

Mort, P.R.: *Scale-up of binder agglomeration processes*, Powder Technology, No. 150, pp. 86-103, **2005**.

Nedderman, R.M.: *Statics and Kinematics of Granular Materials*, Cambridge University Press, Cambridge, **1992**.

Newton , D.: *Future Challenges in Fluidized Bed Technology*, Skandinavisk Teknikförmidling International Ab, Bromma, **1995**.

Ouchiyama, N. and Tanaka, T.: *The probability of coalescence in granulation kinetics*, Industrial & Engineering Chemistry Process Design Development, No.14, pp. 286-289, **1975**.

Pain, C.C., Mansoorzadeh, S. and de Olivera, C.R.E.: *A study of bubbling and slugging fluidised beds suing the two-fluid granular temperature model*, International Journal of Multiphase Flow, No. 27, pp. 527-551, **2001**.

Pierrat, P. and Caram, H.S.: *Tensile strength of wet granular materials*, Powder Technology, No. 91, pp. 83-93, **1997**.
Pietsch, W.: *An interdisciplinary approach to size enlargement by agglomeration*, Powder Technology, No. 130, pp. 8-13, **2003**.

Princen, H.M.: *Comments on "the effect of capillary liquid on the force of adhesion between spherical solid particles"*, Journal of Colloid Interface Science, No. 26 pp. 249, **1968**.

Rabinovich, Y.I., Esayanur, M.S. and Moudgil, B.M.: *Capillary Forces between Two Spheres with a Fixed Volume Liquid Bridge: Theory and Experiment*, Langmuir, No. 21, pp. 10992-10997, **2005**.

Rambali, B., Baert, L. and Massart, D.L.: *Scaling up of the fluidized bed granulation process*, International Journal of Pharmaceutics, No. 252, 197-206, **2003**.

Ramkrishna, D.: *The status of population balances*, Chemical Engineering, No. 3, pp. 49–95, **1985**.

Ramkrishna, D.: *Population Balances – Theory and Applications to Particulate Systems in Engineering*, Academic Press, London, **2000**.

Ramkrishna, D. and Mahoney, A.W.: *Population balance modelling: Promise for the future*, Chemical Engineering Science, No. 57, pp. 595-606, **2002**.

Randolph, A.D. and Larson, M.A.: *Theory of Particulate Processes – Analysis and Techniques of Continuous Crystallization*, Academic Press, NY, **1971**.

Randolph, A.D. and Larson, M.A.: *Theory of Particulate Processes – Analysis and Techniques of Continuous Crystallization*, 2nd Edition, Academic Press, NY, **1988**.

Reynolds, G.K., Fu, J.S., Cheong, Y.S., Hounslow, M.J. and Salman, A.D.: *Breakage in granulation: A review*, Chemical Engineering Science, No. 60, pp. 3969-3992, **2005**.

Rhodes, M.: *Introduction to Particle Technology*, John Wiley & Sons Ltd., Chichester, **1998**.

Rosato, A., Prinz, F., Strandburg, K.J. and Swendsen, R.: *Monte Carlo Simulation of Particulate Matter Segregation*, Powder Technology, No. 49, pp. 59-69, **1986**.

Rumpf, H.: *The strength of Granules and Agglomerates* in W.A. Knepper: *Agglomeration*, American Institute of Mining, Metallurgical, and Petroleum Engineers, INC., Interscience Publishers, NY, pp. 379-418, **1962**.

Samuelsberg, A.E. and Hjertager, B.H.: *Computational fluid dynamic simulation of an oxy-chlorination reaction in a full-scale fluidized bed reactor*, Proceedings of the 5th International Conference on Circulating Fluidized Beds, Beijing, **1996**.

Sanderson, J. and Rhodes, M.: *Bubbling Fluidized Bed Scaling Laws: Evaluation at Large Scales*, Particle Technology and Fluidization, Vol. 51, No. 10, pp. 2686-2694, **2005**.

Sastry, K.V.S.: *Similarity Size Distribution of Agglomerates during their Growth by Coalescence in Granulation or Green Pelletization*, International Journal of Mineral Processing, No. 2, pp. 187-203, **1975**.

Schaafsma, S.H., Vonk, P., Segers, P. and Kossen, N.W.F.: *Description of agglomerate growth*, Powder Technology, No. 97, pp 183-190, **1998**.

Schaafsma, S.H., Vonk, P., and Kossen, N.W.F.: *Fluid bed agglomeration with a narrow droplet size distribution*, International Journal of Pharmaceutics, No. 193, pp. 175-187, **2000a**.

Schaafsma, S.H.: *Down-scaling of a fluidised bed agglomeration process*, Rijkuniversiteit Groningen, **2000b**.

Schouten, J.C., Van der Stappen, M.L.M. and Van den Bleek, C.M.: *Scale-Up of Chaotic Fluidized Bed Hydrodynamics*, Chemical Engineering Science, Vol. 51, No. 10, pp. 1991-2000, **1996**.

Schubert, H.: *Kapillardruck und Zugfestigkeit von feuchten Haufwerken aus kornigen Stoffe*n, Chemie Ingineur Teknik, No. 6, pp. 396-401, **1973**.

Schubert, H.: *Tensile Strength of Agglomerates*, Powder Technology, No. 11, pp.107-119, **1975**.

Schæfer, T. and Wørts, O.: *Control of fluidized bed granulation. I. Effects of spray angle, nozzle height and starting materials on granule size and size distribution*, Archives of Pharmaceutical and Chemical Science, 5th Edition, pp. 51-60, **1977a**.

Schæfer, T. and Wørts, O.: *Control of fluidized bed granulation. II. Estimation of droplet size of atomised binder solutions*, Archives of Pharmaceutical and Chemical Science, 5th Edition, pp. 178-193, **1977b**.

Schæfer, T. and Wørts, O.: *Control of fluidized bed granulation. III. Effects of inlet air temperature and liquid flow rate on granule size and size distribution. Control of moisture content of granules in the drying phase*, Archives of Pharmaceutical and Chemical Science, 6th Edition, pp. 1-13, **1978a**.

Schæfer, T. and Wørts, O.: *Control of fluidized bed granulation. IV. Effects of binder solution and atomization on granule size and size distribution*, Archives of Pharmaceutical and Chemical Science, 6th Edition, pp. 14-25, **1978b**.

Schæfer, T. and Wørts, O.: *Control of fluidized bed granulation. V. Factors affecting granule growth*, Archives of Pharmaceutical and Chemical Science, 6th Edition, pp. 69-82, **1978c**.

Scott Fogler, H.: *Elements of Chemical Reaction Engineering*, 3rd International Edition, Prentice-Hall Inc., Upper Saddle River, NJ, **1999**.

Sheffield. *Population Balance Modelling – Forty years in the Balance*. Commercial flyer, University of Sheffield, **2005**.

Simons, S.J.R. and Seville, J.P.K. and Adams, M.J.: *An Analysis of the Rupture Energy of Pendular Liquid Bridges*, Chemical Engineering Science, Vol. 49, No. 14, pp. 2331-2339, **1994**.

Squires, A.M.: *Contribution toward a history of fluidization*, Adapted from: Proceedings of the Joint Meeting of Chemical Engineering Society of China and American Institute of Chemical Engineers, Chemical Industry Pres, Beijing, pp. 322-353, **1982**.

Sudsakorn, K. and Turton, R.: *Nonuniformity of particle coating on a size distribution of particles in a fluidized bed coater*, Powder Technology, No. 110, pp.37-43, **2000**.

Summers, M. & Aulton, M.: *Pharmaceutics. The Science of Dosage Form Design*, 2nd Edition, Montford University, Churchill Livingstone, Leicester, **2001**.

Sun, D-W.: *Computational fluid dynamics (CFD) – a design and analysis tool for the agri-food industry*, Computers and Electronics in Agriculture, No. 34, pp. 1-3, **2002**.

Taghipour, F., Ellis, N. and Wong, C.: *Experimental and computational study of gas-solid fluidized bed hydrodynamics*, Chemical Engineering Science, No. 60, pp. 6857-6867, **2005**.

Talu, I, Tardos, G.I. and Ruud van Ommen, J.: *Use of stress fluctuations to monitor wet granulation of powders*, Powder Technology, No. 117, pp. 149-162, **2001**.

Tan, H.S., M.J.V. Goldschmidt, Boerefijn, B., Hounslow, M.J., Salman, A.D. and Kuipers, J.A.M.: *Building population balance for fluidized bed granulation: Lessons from kinetic theory of granular flow*, Proceedings from 4th World Congress of Particle Technology in Sydney, **2002**.

Tan , H.S., Salman, A.D. and Hounslow, M.J.: *Kinetics of fluidised bed melt granulation. IV. Selecting the breakage model*, Powder Technology, No. 143-144, pp. 65-83, **2004**.

Tardos, G.I., Khan, M.I. and Mort, P.R.: *Critical Parameters and Limiting Conditions in Binder Granulation of Fine Powders*, Powder Technology, No. 94, pp. 245-258, **1997**.

Teunou, E. and Poncelet, D.: *Batch and continuous fluid bed coating – review and state of the art*, Journal of Food Engineering, No. 53, pp. 325-340, **2002**.

Thornton, C. and Ciomocos, M.T.: *Numerical simulations of agglomerate impact breakage*, Powder Technology, No. 105, pp. 74-82, **1999**.

Tsuji Y., Kawaguchi T. and Tanaka T.: *Discrete particle simulation of two-dimensional fluidized bed*, Powder Technology, No. 77, pp. 79-87, **1993**.

Turchiuli, C., Eloualia, Z., Mansouri, N.E. and Dumoulin, E.: *Fluidised bed agglomeration: Agglomerates shape and end-sue properties*, Powder Technology, No. 157, pp. 168-175, **2005**.

Turton, R. and Cheng, X.X.: *The scale-up of spray coating processes for granular solids and tablets*, Powder Technology, No. 150, pp. 78-85, **2005**.

van den Bleek, C.M. and Schouten, J.C.: *Deterministic chaos: a new tool in fluidized bed design and operation*, The Chemical Engineering Journal, No. 53, pp. 75-87, **1996**.

van der Stappen, M.L.M.: *Chaotic Hydrodynamics of Fluidized Beds*, Ph.d. Thesis, Technische Universiteit Delft, **1996**.

Verkoeijen, D., Pouw, G.A., Meesters, G.M.H. and Scarlett, B.: *Population balances for particulate processes – a volume approach*, Chemical Engineering Science, No. 57, pp. 2287-2303, **2002**.

Waldie, B.: *Growth mechanism and the dependence of granule size on drop size in fluidized-bed granulation*, Chemical Engineering Science, No. 46, pp. 2781-2785, **1991**.

Wang, L. and Sun, D-W.: *Recent developments in numerical modelling of heating and cooling in the food industry – a review*, Trends in Food Science & Technology, No. 14, pp. 408-423, **2003**.

Wang, F.Y., Ge, X.Y., Balliu, N. and Cameron, I.T.: *Optimal control and operation of drum granulation processes*, Chemical Engineering Science, No. 61, pp. 257-267, **2006**.

Watano, S., Sato, Y., Miyanami, K., Murakami, T., Ito, Y., Kamata and Oda, N.: *Scale-Up of Agitation Fluidized Bed Granulation I. Preliminary Experimental Approach for Optimization of Process Variables*, Chemical and Pharmaceutical Bulletin, No. 43, pp. 1212-1216, **1995a**.

Watano, S., Sato, Y. and Miyanami, K.: *Scale-Up of Agitation Fluidized Bed Granulation IV. Scale-Up Theory Based on the Kinetic Energy Similarity*, Chemical and Pharmaceutical Bulletin, No. 43, pp. 1227-1230, **1995b**.

Wauters, P.A.L.: *Modelling and Mechanisms of Granulation*, Ph.d. Thesis, Technische Universiteit Delft, **2001.**

Wauters, P.A.L., Jakobsen, R.B., Litster, J.D., Meesters, G.M.H. and Scarlett, B.: *Liquid distribution as a means to describe the granule growth mechanism*, Powder Technology, No. 123, pp. 166-177, **2002**.

Webopedia. Visit on the homepage www.webopedia.com, December **2005**.

Weinbaum, S. and Caro, C.G.: *A macromolecule transport model for the arterial wall and endothelium based on the ultrastructural specialisation observed in electron microscopic studies*, Journal of Fluid Mechanics, No. 74, pp. 611 – 640, **1976**.

Werther, J.: *Modelling and Scale-up of Industrial Fluidized Bed Reactors*, Chemical Engineering Science, No. 35, pp. 372-379, **1980**.

Wikipedia. Visit on the homepage www.wikipedia.org, November **2005**.

Wildeboer, W.J., Litster, J.D. and Cameron, I.T.: *Modelling nucleation in wet granulation*, Chemical Engineering Science, No. 60, pp. 3751-3761, **2005**.

Willet, C.D., Adams, M.J., Johnson, S.A. and Seville, J.P.K.: *Capillary Bridges between Two Spherical Bodies*, Langmuir, No. 16, pp. 9396-9405, **2000**.

Wisconsin. University of Madison-Wisconsin. Visit on the homepage: www.wisc.edu, December **2005**.

Xia, B. and Sun D-W.: *Applications of computational fluid dynamics (CFD) in the food industry: a review*, Computers and Electronics in Agriculture, No. 34, pp. 5-24, **2002**.

Xiong, Y. and Pratsinis, S.: *Formation of agglomerate particles by coagulation and sintering—Part I. A two-dimensional solution of the population balance equation*, Journal of Aerosol Science, No. 24, pp. 283–300, **1993**.

Yates, J.G.: *Fundamentals of Fluidized-bed Chemical Processes*, Butterworths Monographs in chemical Engineering, Butterworths, London, **1983**.

York, D.W.: *An industrial user's perspective on agglomeration development*, Powder Technology, No. 130, pp. 14-17, **2003**.

Notes

- [i] That is, continuous variables.
- [ii] The particle state vector does not necessarily have to be finite but can in some problems be an infinite dimensional state vector, which sometimes can be made finite using e.g. Fourier series expansion (Ramkrishna, 2000).
- [iii] In case nothing else is stated, the theory in section 1.2 is adapted from Ramkrishna (2000).
- [iv] Note that as the vector **x** only contains one element, the symbol **x** is replaced by the scalar x.
- [v] As it will appear in the boundary conditions for the derived population balance, it may be assumed that the present granulation process involves initial nucleation resulting in rudimentary particles with size and mass equal to zero, which then subsequently grow according to G(x,t). The introduction of a nucleation rate for x = 0 does, however, not conflict with the assumption of neglecting birth and death of particles in interval [a,b].
- [vi] In this case aggregation should be understood as coalescence. Traditionally, articles concerning population balances prefer aggregation as this term covers a variety of processes ranging from coalescence to coagulation (Randolph & Larson, 1971 & 1988 and Ramkrishna, 2000).
- [vii] More firmly it is assumed that the probability of more than two particles aggregating simultaneously to form a single particle is of the order of $O(dt^2)$ while that of two particles aggregating (binary aggregation) is of the order of O(dt) (Ramkrishna, 2000).
- [viii] Adetayo & Ennis (1997) stated that the cut-off size in their kernel was expected to vary with granule and binder properties. However, none of the material and binder properties was used in their population balance model.
- [ix] Adetayo & Ennis (1997) assumed the parameters a_{AE} and b_{AE} would vary with granule deformability and allowed them to vary to fit to granulation data. Note that to be dimensional consistent it is required that
- $2b_{AE} - a_{AE} = 1$ (Litster & Ennis, 2004).
- [x] Non-uniform component distribution during granulation is a common problem (Sudsakorn & Turton, 2000 and Iveson, 2002). In e.g. solid enzyme production the enzyme molecules are much smaller than the carrier particles. When mixtures of these "powders" are co-granulated it is frequently noted that a disproportionate amount of enzyme component is found in the fines fraction. Hence it is relevant in enzyme granulation as well as in most pharmaceutical applications to account for the differences in composition between granules.
- [xi] Also known as the mean value theorem on frequency (Wang et al., 2006).
- [xii] The original Hounslow model is hereby made a special case in which q=1 (Cryer, 1999).
- [xiii] For Monte Carlo analysis combined with population balances, Cryer & Scherer (2003), Wauters (2001) and Madec et al. (2003) should also be consulted.